GUIDELINES for CH 7.2 INTEGRALS.

① $\int \cos^m(x)\,dx$ — OR SINE. (REVERSE ROLL of SINE and COSINE IN PROCEDURE)

- **m EVEN:** Write as $(\cos^2(x))^{\square}$ (POWER $m/2$) and use Half Angle identities. May need to repeat!
- **m ODD:** Write as $\cos^{m-1}(x)\cdot\cos(x)$. Since $m-1$ will be even, we then write $(\cos^2(x))^{\square}\cdot\cos(x)$ (POWER $\frac{m-1}{2}$; "STRAGGLER"). Next use Pythagorean Identity $\cos^2(x)=1-\sin^2(x)$ to convert everything to SINE (except for the straggler) and complete with the method of substitution using: $w=\sin(x)$

② $\int \cos^m(x)\sin(x)\,dx \longrightarrow$ Let $w=\cos(x)$ and complete with the method of substitution.

③ $\int \sin^m(x)\cdot\cos(x)\,dx \longrightarrow$ Let $w=\sin(x)$ and complete with the method of substitution.

④ $\int \sin^m(x)\cos^n(x)\,dx$ for $m,n>1$

Three cases:

- If n is ODD, then $n=2k+1$. Pull out one COSINE to rewrite as $\sin^m(x)(\cos^2(x))^k\cdot\cos(x)$ ("STRAGGLER"). Then use the Pythagorean Identity $\cos^2(x)=1-\sin^2(x)$ to rewrite remaining factors in terms of SINE. Finally, use the method of substitution with $w=\sin(x)$ which will eliminate straggler.
- If m is ODD, then $m=2k+1$. Pull out one SINE to rewrite as $(\sin^2(x))^k\cdot\sin(x)\cdot\cos^n(x)$ ("STRAGGLER"). Then use the Pythagorean Identity $\sin^2(x)=1-\cos^2(x)$ to rewrite remaining factors in terms of COSINE. Finally, use the method of substitution with. $w=\cos(x)$ which will eliminate straggler
- If both m and n are ODD, follow procedure for the <u>smaller</u> odd value (it will work either way but this will save time).
- If both m and n are EVEN. Convert everything to either SINE or COSINE using Half Angle Identities. It is oftentimes helpful to use the double angle identity $\sin(2x)=2\sin(x)\cos(x)$

EXAMPLES

Let's check out a few more integrals so that we can see how these "guidelines" apply.

Ex 1. A

$$\int \sin^4(x)\,dx = \int (\sin^2(x))^2\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$$

m EVEN

HALF ANGLE

EXPAND.

GUIDELINE 1A.

$$= \frac{1}{4}\int \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx$$

HALF ANGLE

$$= \frac{1}{4}\int \left(1 - 2\cos(2x) + \frac{1+\cos(4x)}{2}\right) dx =$$

$$= \frac{1}{4}\int \left(\tfrac{3}{2} - 2\cos(2x) + \tfrac{1}{2}\cos(4x)\right) dx = \boxed{\frac{1}{4}\left(\tfrac{3}{2}x - \sin(2x) + \frac{\sin(4x)}{8}\right) + C}$$

SUBST. $W = 2X$ NEEDED

SUBST. $W = 4X$ NEEDED.

B

REWRITE

$$\int \cos^3(x)\,dx = \int \cos^2 x \cos(x)\,dx$$

m ODD

PYTH. IDENT

GUIDELINE 1B.

$$= \int (1 - \sin^2 x)\cos(x)\,dx$$

$$= \int (1 - w^2)\,\cancel{\cos(x)}\,\frac{dw}{\cancel{\cos(x)}}$$

$$\boxed{\begin{aligned} w &= \sin(x) \\ dw/dx &= \cos(x) \\ dx &= \frac{dw}{\cos(x)} \end{aligned}}$$

$$= \int 1 - w^2\,dw = w - w^3/3 + C$$

$$= \boxed{\sin(x) - \frac{\sin^3(x)}{3} + C}$$

C

$$\int \sin^7(x)\cos(x)\,dx = \int (\sin(x))^7 \cos(x)\,dx$$

GUIDELINE 3.

$$= \int (w)^7 \cdot \cancel{\cos(x)}\,\frac{dw}{\cancel{\cos(x)}}$$

$$\boxed{\begin{aligned} w &= \sin(x) \\ dw/dx &= \cos(x) \\ dx &= \frac{dw}{\cos(x)} \end{aligned}}$$

$$= \int w^7\,dw =$$

$$w^8/8 + C = \boxed{\frac{\sin^8(x)}{8} + C}$$

D $\int \cos^5(x)\sin^3(x)\,dx = \int \cos^5(x)\cdot\underbrace{\sin^2(x)}_{\text{PYTH. THM.}}\sin(x)\,dx$

REWRITE

GUIDELINE 4.

$$= \int \cos^5(x)\cdot(1-\cos^2(x))\cdot\sin(x)\,dx$$

$$= \int w^5(1-w^2)\cdot\cancel{\sin(x)}\,\frac{dw}{-\cancel{\sin(x)}}$$

$$= -\int w^5(1-w^2)\,dw = -\int w^5 - w^7\,dw$$

$$= -\frac{w^6}{6} + \frac{w^8}{8} + C$$

$$\boxed{= -\frac{\cos^6(x)}{6} + \frac{\cos^8(x)}{8} + C}$$

$w = \cos(x)$

$dw/dx = -\sin(x)$

$dx = \dfrac{dw}{-\sin(x)}$