Implementing Online Programs in Gateway Mathematics Courses for Students with Prerequisite Deficiencies

Elliott Bertrand, David T. McArdle, Lubos Thoma & Li Wu

To cite this article: Elliott Bertrand, David T. McArdle, Lubos Thoma & Li Wu (2021) Implementing Online Programs in Gateway Mathematics Courses for Students with Prerequisite Deficiencies, PRIMUS, 31:2, 119-132, DOI: 10.1080/10511970.2019.1629556

To link to this article: https://doi.org/10.1080/10511970.2019.1629556

Published online: 09 Aug 2019.
Implementing Online Programs in Gateway Mathematics Courses for Students with Prerequisite Deficiencies

Elliott Bertrand, David T. McArdle, Lubos Thoma, and Li Wu

ABSTRACT

The percentage of college students receiving unproductive grades in introductory mathematics courses is a concern for post-secondary institutions across the country. Many factors can be targeted as potential explanations for this lack of success, yet none of these issues are more noteworthy than the fact that many students enter college mathematics courses with significant gaps in their fundamental mathematical background. In this paper, the authors discuss a way to implement an online remedial program to help students overcome their deficiencies following a “just-in-time teaching” model. Relevant data are presented supporting a positive outcome.

KEYWORDS

Mathematics; remedial mathematics; online; just-in-time teaching; prerequisites

1. INTRODUCTION

Mathematics forms the foundation for many academic disciplines ranging from engineering and science to business and economics. It is vital that students preparing for careers in these fields have a strong mathematical background in order to successfully complete their degrees. Consequently, a great deal of attention has been devoted to student success in introductory mathematics courses.

Introductory mathematics courses are generally classified as gateway courses. A gateway course involves high enrollment, has a high unproductive rate (that is, the percentage of students either earning a D, F, or incomplete or withdrawing), and is foundational in the sense that it provides students with basic knowledge necessary for their future studies. Unsurprisingly, it has been found that students’ levels of success in gateway courses will ultimately determine the scope of their academic career; see [6] and [12].

Two of the most prevalent gateway mathematics courses are precalculus and calculus, which both have alarmingly high unproductive rates nationwide. The unproductive rates for these courses at the University of Rhode Island during the academic years between 2013 and 2017 can be seen in Figures 4 and 5. These high unproductive rates have profound consequences, and thus it is important to identify the causes for the lack of student success and make necessary pedagogical
adjustments. For a discussion of pedagogical changes that have been made recently, see [2], [4], and [11].

The authors in [5] identified the major factors affecting student achievement in mathematics, and it was found that a student’s prerequisite knowledge was one of the most prominent. From an educator’s perspective, it is often necessary and beneficial to reinforce basic mathematical content to account for any prerequisite deficiencies that may exist. An effective way to do this involves a just-in-time teaching (JiTT) approach where students are provided with remediation exactly when it is needed [10]. In [7] and [9], the authors use a JiTT approach by embedding review into a calculus course. A similar approach is used in [8], where the authors incorporate online review homework assignments into the calculus curriculum to ensure that students are re-exposed to necessary content at opportune times. This method can also be seen in a slightly different setting within [1] and [13].

At the University of Rhode Island, the Department of Mathematics sought to improve the mathematical preparedness of students within precalculus and calculus courses by implementing a structured, online review environment following a JiTT structure. The intention was to improve on the methods from [1], [7], [8], [9], and [13] by creating an on-demand, fully online remedial program that would not involve a reduction in enrollment capacity, would not require valuable class time to facilitate, and would not delay a student’s academic course progression. This led to the development and implementation of the CAE (College Algebra Enhancement) and PCE (Precalculus Enhancement) Projects, which constitute the main focus of the current article.

2. CONSTRUCTION AND DESIGN

The CAE Project and the PCE Project have a primary goal of helping students at the University of Rhode Island improve their mathematical preparedness for precalculus and calculus, respectively. The projects each consist of five or six weekly lessons that are hosted on the university’s classroom management system, Sakai. Each lesson focuses on several prerequisite topics and includes micro-video lectures, review problems, lesson highlights, and online quizzes. Although the ultimate goal is to assist the students in understanding their current studies, the focus of the projects is on prerequisite skills. The intention is for students to become reacquainted with important concepts in a “vacuum” environment by purely focusing on relearning essential skills. After completing the project, students can then apply their polished skills to relevant topics within their gateway course.

In the coming sections, specific details involving the design and implementation of the CAE Project, in particular, are provided.

2.1. Linking Prerequisite Skills to Topics in the Gateway Course

A great deal of material covered in a gateway curriculum is naturally dependent upon skills learned in a recent course. The foundation of the CAE (and PCE)
Table 1. Examples of prerequisite skills needed for precalculus.

<table>
<thead>
<tr>
<th>Prerequisite Skill for Project</th>
<th>Associated Gateway Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order of Operations</td>
<td>Function Evaluation</td>
</tr>
<tr>
<td>Simplifying Expressions</td>
<td>Solving Linear Equations</td>
</tr>
<tr>
<td>Function Notation</td>
<td>Difference Quotient</td>
</tr>
<tr>
<td>Exponent Rules</td>
<td>Inverse Functions</td>
</tr>
<tr>
<td>Rational Exponents</td>
<td>Inverse Functions</td>
</tr>
<tr>
<td>Adding and Subtracting Polynomials</td>
<td>Operations with Functions</td>
</tr>
<tr>
<td>Multiplying Polynomials</td>
<td>Operations with Functions</td>
</tr>
<tr>
<td>Dividing Polynomials</td>
<td>Operations with Functions</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>Solving Quadratic Equations</td>
</tr>
<tr>
<td>Greatest Common Factor</td>
<td>Solving Polynomial Equations</td>
</tr>
<tr>
<td>Factoring by Grouping and Factoring Trinomials</td>
<td>Solving Polynomial Equations</td>
</tr>
<tr>
<td>Simplifying, Adding, and Subtracting Radical Expressions</td>
<td>Solving Polynomial Equations</td>
</tr>
<tr>
<td>Multiplying and Dividing Radical Expressions</td>
<td>Solving Polynomial Equations</td>
</tr>
<tr>
<td>Factoring Special Products</td>
<td>Solving Polynomial Equations</td>
</tr>
<tr>
<td>Solving Equations by Factoring</td>
<td>Solving Polynomial Equations</td>
</tr>
<tr>
<td>Adding and Subtracting Rational Expressions</td>
<td>Rational Functions</td>
</tr>
<tr>
<td>Multiplying and Dividing Rational Expressions</td>
<td>Rational Functions</td>
</tr>
<tr>
<td>Solving Rational Equations</td>
<td>Rational Functions</td>
</tr>
</tbody>
</table>

Project is the effective identification of these specific prerequisite skills. We used historical evidence through both informal observation and formal analysis of data to target these particular concepts. In essence, the projects work to more closely align the objectives of a gateway course with those of its predecessors. This can prove challenging in a college freshman course, since incoming students have varied backgrounds that make it difficult to pinpoint their level of experience. The lack of consistency in past education does itself reinforce the need for such remedial programs.

The CAE Project specifically targets prerequisite algebra skills needed for success in a precalculus course. See Table 1 for examples of the skills that were deemed essential in this context. Once the prerequisite topics were identified, we linked each topic with essential concepts from the gateway course. Many of them, as should be expected, naturally align with relevant precalculus material. For example, facility with function notation is critical in precalculus when composing functions or calculating the difference quotient. Table 1 illustrates the specific links that were made between each prerequisite algebra skill and the precalculus topics for which the algebra skill is relevant. We have provided one example (perhaps the first on which the gateway course may focus chronologically) for each identified prerequisite skill. The gateway skills listed are not exhaustive, but they are among the most relevant.

It is important to note that while incoming students of a gateway course may have some recollection of prerequisite material, these skills often need refinement or reinforcement. This is precisely the purpose of the remedial projects.

2.2. Dividing the Prerequisite Skills into Weekly Lessons

After we identified specific prerequisite skills, we organized them into weekly lessons through which they were presented to students. The total length of the
project accommodates the pace of the accompanying gateway course and the number (and difficulty) of skills involved. In our precalculus course we identified 18 remedial skills to be covered in the project. After setting the main project length to five weeks, we then allocated three or four topics for each weekly lesson. We grouped the prerequisite skills into weekly lessons such that they complement each other in substance. For instance, the skills of adding and subtracting rational expressions fit in well with the skills of multiplying and dividing rational expressions, so we cover these skills in the same lesson.

Most importantly, the skills were ordered chronologically based on when their associated gateway skills are covered in the course. This organization is essential to ensure the project follows a JiTT model. The aim of the project is to reintroduce essential prerequisite skills to students just before (or as) they are about to utilize these skills in their gateway course. In this way, we prepare students to be proactive and not reactive; students are prepared for upcoming concepts and can proceed with confidence in their gateway course.

2.3. Constructing the Lesson Content

After designing a lesson timeline, we created the content to be presented in the online lessons. There are four main resources that are provided in each lesson: micro-videos, “Lesson Highlights,” “Test Yourself” practice problems, and pre-quizzes/project quizzes.

2.3.1. Micro-videos

Since this project operates entirely online, micro-video lectures are perhaps the most essential component. In fact, in a survey of students, 39% communicated that they found the videos to be the most valuable part of the online program. Each micro-video focuses on precisely one prerequisite skill and generally lasts between 5 and 10 minutes, which helps to procure greater attention and viewership. Since this is a remedial project, students should have prior experience with the topics covered in the lessons, and thus, the videos can be made more concise without sacrificing effectiveness.

In these videos, we simply present the facts. It is not within the purview of the videos to introduce the material formally and define every term explicitly. We seek only to present one prerequisite skill in a given video; we clearly introduce the concept and then present relevant material with examples. Software such as Explain Everything™ or Camtasia® provided exactly the tools that were needed to produce a polished and effective micro-video. A still image of an example video can be seen in Figure 1. For a discussion of the effectiveness of instructor-generated videos with regard to student learning, see [3].

2.3.2. Lesson Highlights

The micro-videos are designed to be brief, but they still may cover quite a bit of material. Moreover, although the micro-video may appeal more to the aural learner,
Figure 1. A micro-video on factoring special products.

an additional tool may better appeal to the visual learners: the “Lesson Highlights.” In each Lesson Highlights, we present a distillation of each prerequisite skill by providing step-by-step instructions on how to utilize or apply a certain skill within a specific problem or application. The key here is reinforcement; this tool literally highlights the essential components in applying a prerequisite skill. We require¹ that students watch the videos and consult the highlights before proceeding to the assessment portion of the lesson. Students may use the Lesson Highlights as a manual as they continue practicing relevant skills, and they may also use the document as effective review before they move on in the lesson. For an example of Lesson Highlights, see Figure 2.

2.3.3. Test Yourself practice
After watching all of the micro-videos for the week’s lesson and reviewing the Lesson Highlights, a student is directed to practice. For each lesson, a document is provided that contains multiple practice problems incorporating the skills on which students were briefed in the videos. We simply provide illustrative examples to the students and encourage them to attempt these problems at their own pace. As the name “Test Yourself” suggests, this component of the lesson facilitates self-guided practice. The students participating in the program use this component as a concentrated, focused area in which to try additional practice problems. Students can also utilize this resource prior to starting the lesson to gauge their level of understanding and guide their participation in the lesson.

¹ Course management software such as Sakai allows the course creator to require students to visit one resource by clicking on the corresponding link before continuing on with the remainder of the lesson. Such functionality is especially useful in organizing the pace of students’ completion of the project.
2.3.4. Pre-quizzes and project quizzes

After students have watched the micro-videos, completed sufficient practice problems, and reviewed the Lesson Highlights, they are assessed on their understanding of the skills covered in the weekly lesson. We introduce two types of assessment in each lesson: pre-quizzes and project quizzes. Each lesson contains one pre-quiz that consists of 10–20 questions that correspond to the lesson’s skills. Students are allowed unlimited submission attempts for the pre-quiz, and no submission counts toward their overall project score. However, each student is required to obtain a satisfactory score (typically 60%) before being allowed to progress to the project quiz. The project quiz is the only graded component of each lesson, and the scores from all project quizzes are totaled to form the student’s project score. This assessment mirrors the structure of the corresponding pre-quiz, is timed, and allows for only one submission attempt. The paired pre-quiz and project quiz draw questions from a common pool, so we have created numerous and varied questions that assess a student’s facility with a given skill from multiple perspectives. Since the students must complete the pre-quiz before they move on, they are free to do so only when they are confident in their skills; in this way, the assessment component of this project is self-guided and may be taken at each student’s own pace.

2.4. Motivating Student Participation in the Project

The final component of implementing the online remedial projects involves motivating students to participate in the project. In order to do this, we first make the project a course requirement that is included in the syllabus of the gateway.
Some percentage (about 5–10%) of the final course grade is reserved for the students’ successful completion of the remedial project (or a passing grade on a diagnostic test). Students take a diagnostic test during the first week of classes that is designed to measure their comprehension of the essential prerequisite skills discussed in Section 2.1. Those students who score a 70% or above are exempted from the project and are given full marks for the corresponding component of their course grade. Students who do not satisfactorily pass the diagnostic test are automatically enrolled into the online remedial project. Participants who gain a 70% or above in the project earn full marks towards the corresponding component of their course grade, whereas those who score below this threshold earn a prorated percentage of the points towards their final grade. See Figure 3 for the student welcome page explaining this protocol.

In addition to the remedial project being tied into the gateway course’s grading breakdown, the intention is to also demonstrate to students that the project can be of great benefit to them. During the semester, the project isolates the prerequisite material, so the student will not see the direct benefit at that point. However, in the course itself the instructors are encouraged to draw parallels to the material being reinforced in the remedial project. The instructor does not need to refresh the students’ memory of these skills, since this is exactly the purpose of the accompanying project, but the instructor should verbally draw the connection so that the students

---

**Figure 3.** The CAE’s welcome page.
can understand the underlying benefit of reviewing the prerequisite material. The topics identified to be studied in the remedial project are important in the sense that they are directly relevant to material in the gateway course. As a result, the associated gateway skills are listed as course objectives in the syllabus. This provides concrete evidence supporting the importance of the skills learned in the course and those discussed in the remedial project.

3. EVALUATION AND RESULTS

Since the implementation of the CAE and PCE remedial projects at the University of Rhode Island (URI), the Department of Mathematics has collected data related to their overall effectiveness. These data include official course unproductive rates as well as students’ perceptions of the projects. It was found that the unproductive rates within the affected courses decreased and students had overwhelmingly positive feedback regarding the projects.

Figure 4 displays the unproductive rates for MTH 111 (Precalculus) at URI from the academic semesters between 2013 and 2017. The CAE Project has been active since the spring semester of 2015, and a noticeable improvement in unproductive rates has been observed. Figure 5 shows results for the PCE Project in MTH 131 (Applied Calculus I), which was first implemented in the fall semester of 2013. More compelling results can be seen by differentiating between the students who were required to participate in the CAE (or PCE) Project and those who were exempt and tracking the resulting unproductive rates. For the students who were required to participate, one can also differentiate between those who demonstrated proficiency in the program and those who did not. These data are displayed in Figures 6 and 7 for the CAE Project. The students who participated in the CAE Project, and demonstrated proficiency, had a 31% chance of receiving an unproductive grade for the course, whereas students who were exempt from the project had a 35% chance of
Moreover, the CAE Project proved to be an effective indicator of students’ levels of preparedness to move on to the next course in the calculus sequence. In fact, 84% of students who participated in the project, but did not demonstrate proficiency, went on to receive an unproductive grade for the course.

In addition to the improvements that were made with regard to the unproductive rates in the affected courses, the CAE and PCE programs were also well-received.
among the students who participated. At the conclusion of the CAE Project, students were asked to choose the option that best reflected their perception of the project from a list of four pre-written responses:

(A) This project was excellent and will help me to pass MTH 111 with a much higher grade than expected.
(B) This project helped me review many algebra skills.
(C) This project helped me review a few algebra skills.
(D) This project is not very helpful and it does not offer as much help as I expected.

The distribution of their responses can be seen in Figure 8. It was found that 64.89% of students rated the project as excellent or good, whereas less than 5% of students found the project to be of no use to them in the gateway course. Similar results were observed for the PCE Project.

Students were also given an opportunity to provide comments about their experiences in the CAE Project, and many found it to be a valuable refresher. Some specific comments are included below:

I enjoyed this program and I am glad that it was available to me. I was able to sharpen my algebra skills, which helped me understand my Pre-Calc homework. Thank you for this opportunity.

I found the CAE Project to be very helpful. I am a returning student, and I hadn't been in a math class since 2005. My algebra skills were very rusty and this program helped me “refresh” them without having to repeat College Algebra.

Students also recognized that the projects benefited their performance in the gateway course in which they were enrolled.

Overall, the CAE Project was very helpful and prepared me ahead of time for the lessons learned in class.
In order to help determine the effectiveness of each aspect of the lesson, students were also asked to identify their favorite aspect of each lesson by choosing between videos, “Test Yourself” practice problems, pre-quizzes, and Lesson Highlights. Their responses for the CAE Project can be seen in Figure 9. It was found that 76% of students felt that the videos or pre-quizzes were the most helpful lesson features. These data will guide the future modifications that will be made to the project.

4. DISCUSSION AND CONCLUSION

Although our programs have proven to be successful in improving students’ adeptness with prerequisite skills, we have found a few areas that could benefit from improvements (or at least increased awareness and supervision). First, we have emphasized the importance of immediately and efficiently informing the students of
their responsibilities within the project. In the past we tried scheduling an evening “introductory seminar” for our precalculus courses during which important facets of the coordinated syllabus, including the CAE Project, were introduced to the students before taking their initial diagnostic test. Although the basic tenets of the project are listed on the course syllabus and a detailed explanation of the project layout is emailed to participants within the first week, it occurs to us that a hard copy of this information (in the form of an “introductory brochure”) would be useful to provide at the outset of the project. Additionally, it may be useful to incorporate a welcome video and/or a “practice quiz” into the introduction of the project. The implementation of such initiatory tools could expedite students’ facility with the project organization.

Communication is key in this project; although we remind students at the beginning and end of each weekly lesson to complete their remedial lessons, they often forget to submit their work. Some students have suggested that we send out even more email correspondence to participants. Perhaps it could be arranged to notify only the students who have not yet submitted their project quizzes so as to not bother those who have already dutifully submitted their work. In keeping with the goal of increased communication, students should be encouraged to ask questions about lesson material. In our experience, students very rarely ask questions about the review material, but establishing a clearer communication channel may assist in creating a more productive dialogue.

We have also ruminated on the central goal of the project, to remediate prerequisite skills following a JITT model, and considered that the tie-in between the project and the course itself could be more explicitly articulated. Although we encourage the prerequisite skills to be reviewed in an environment initially free of application, we believe the students could benefit from a greater connection to the material presented within the gateway course. To remedy this potential disconnect, we propose that instructors of the gateway course could take a more participatory role in connecting lecture topics to the skills being reviewed in the online project. Additionally, we have considered potentially adding a component of the online project that finally incorporates the prerequisite skills into applications of course material.

Although we will continue to investigate potential improvements that could be made to the projects in the future, we are also aware of the potential to adapt these projects to other settings. A third program, the AEP (Algebra Enhancement Project), has already been created and implemented in college algebra courses at URI to help remediate basic algebra skills. The intention is to create comparable programs to be paired with all high-enrollment freshman- and sophomore-level mathematics courses. These new programs would follow an identical structure to that outlined in the preceding sections, and the hope is that adding such programs will significantly improve unproductive rates across the entire sequence of mathematics courses offered at URI. Moreover, it is important to note that the necessity of remediating prerequisite knowledge is not unique to mathematics. Many content areas, including chemistry, physics, and biology, follow a definite sequence, and
the strength of students’ foundational knowledge greatly affects their future success. Online remedial projects similar to the PCE and CAE Projects can certainly be implemented in these fields to ensure that a student’s prerequisite knowledge is at an appropriate level.

Outside of course remediation, the program structure can also be used to prepare students for a given course prior to enrollment. URI has adapted the CAE and PCE Projects to create summer modules that are completed by incoming freshmen on a voluntary basis. These modules allow interested students to review basic mathematical content that they learned in the past in preparation for a proctored placement exam administered during their orientation. Paired with an appropriate placement policy, this summer module structure can ensure that incoming freshmen begin their college mathematical education at an appropriate level. The modules were offered at URI during the summers of 2016 and 2017, and initial data suggest that participants were able to improve their placement and earn successful marks in gateway courses.

With a growing concern over unproductive rates in mathematics courses and with the widespread effect that mathematics has on other disciplines, it is incredibly important to work to improve student success in mathematics. URI has dedicated a great deal of time and resources to address this issue by implementing programs that can help students overcome prerequisite deficiencies. The remedial programs discussed in this paper do not require a reduction of student enrollment and do not delay the trajectory of a student’s mathematical career. They allow students to stay on track within their intended major, which can greatly improve university-wide 4-year completion and retention rates. With more programs being added annually and refinements being made to existing initiatives, URI hopes to continue to improve student success within mathematics courses in the future.

ACKNOWLEDGEMENTS

The authors would like to thank Laura Barnes for her cooperation in supporting the implementation of the CAE Project in precalculus courses during her tenure as course coordinator at URI. Her enthusiasm for improving students’ comprehension of prerequisite skills was essential to this project’s success. The authors would also like to thank James Baglama, Chair of the Department of Mathematics at URI, for his support of these projects and his help with the preparation of data.

ORCID

Elliott Bertrand http://orcid.org/0000-0001-6037-3501
David T. McArdle http://orcid.org/0000-0002-1632-4857

REFERENCES


**BIOGRAPHICAL SKETCHES**

Elliott Bertrand earned his Ph.D. in mathematics from the University of Rhode Island and is currently an assistant professor at Sacred Heart University in Fairfield, CT. His research focuses on the global dynamics of difference equations, and he is also interested in introducing effective remediation in introductory mathematics courses.

David McArdle earned his Ph.D. in mathematics at the University of Rhode Island in 2017 and is now an assistant professor in residence at the University of Connecticut. His primary research is in the field of discrete dynamical systems but he also has a strong interest in improving mathematics education at the collegiate level.

Lubos Thoma is an associate professor of mathematics at the University of Rhode Island. His current area of research is discrete mathematics. Thoma received his Ph.D. in mathematics from Emory University.

Li Wu is a professor of mathematics at the University of Rhode Island. Her current area of research is numerical analysis. She received her Ph.D. in mathematics from the University of Wyoming.