CH 9.3 SEPARATION of VARIABLES

GOAL: Now that we are familiar with the terminology of DIFFERENTIAL EQUATIONS, we can learn one the main techniques that can be used to actually solve a differential equation: SEPARATION OF VARIABLES.

PART 1: WHAT IS A SEPARABLE DIFFERENTIAL EQ?

* Our focus in this section will be on first-order differential equations (I.e. DEs with only a first derivative) that have an important property:

DEFN: [SEPARABLE DE]:

Ex. Which of the following differential equations are SEPARABLE?

A

D

B

E

C

F

PART 2: THE METHOD of SEPARATION of VARIABLES

STEP 1:

STEP 2:

STEP 3:

Ex 2. Find the general solution to the following differential equations using
SEPARATION OF VARIABLES

A $\frac{dy}{dx} = \frac{x^2}{y^2}$

Sol:

B $(y + xy)y' = 2$

Sol:

C $\frac{dy}{dx} = \frac{2x^2}{y+\sin(y)}$

sol:

D $\frac{dy}{dt} = t^2 y$

sol:

E $xy' + 4 = y^2$

sol:

Ex3: Find the particular solution of each differential equation that satisfies the given initial condition (i.e. solve each **INITIAL VALUE PROBLEM**)

A $\frac{dy}{dx} = \frac{x^2}{y^2}$, $y(0) = 4$

sol:

B $dy/dx = (y + xy)\,y' = 2$

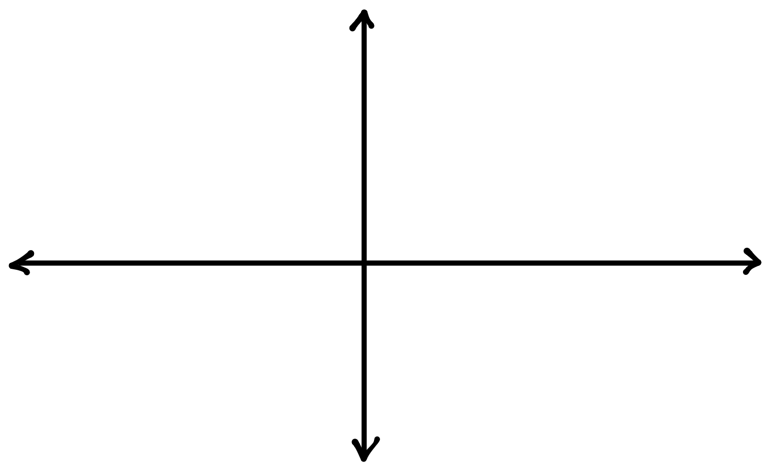
sol:

PART 3: ORTHOGONAL TRAJECTORIES

** Given a family of solution curves that satisfy a differential equation. We will find another collection of curves that are perpendicular to the original family. These new curves are called ORTHOGONAL TRAJECTORIES.

DEFN: [ORTHOGONAL TRAJECTORY]

Some APPLICATIONS



PROCEDURE TO FIND ORTHOGONAL TRAJECTORIES

Ex 4. Find the ORTHOGONAL TRAJECTORIES of the family of curves given by:

A $y = kx^3$

Sol:

B $y = ke^{2x}$

Sol:

PART 4: APPLICATIONS

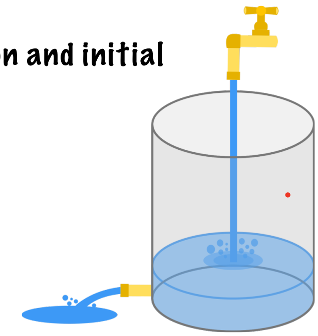
* Let's look at a few more applications of differential equations. We will determine appropriate differential equations to model certain situations and we will then be able to solve them!

NOTE:

Ex 5. A tank contains 30 lbs of sugar dissolved in 200 gallons of water. A solution that contains 0.04 lbs of sugar per gallon of water is pumped into the tank at a rate of 20 gallons/min. The solution is the tank is kept thoroughly mixed, and is drained at the same rate (20 gallons/min). Let y(t) represent the amount of sugar (in lbs) in the tank after t minutes.

[A] Write an IVP that models this situation (i.e. a differential equation and initial condition)

sol



[B] Solve the DE from part [A]

sol:

[C] How much sugar is in the tank after 20 minutes?

sol: