CH 9.1: MODELING w/ DIFFERENTIAL EQ's

GOAL: We will learn what a DIFFERENTIAL EQUATION (DE) is and will explore some basic properties of them. We will also learn how DEs can be used to model certain real world situations.

PART 1: BASICS of DEs

DEFN: [DIFFERENTIAL EQ]:

Ex 1: The following are differential equations

A

B

NOTE: The ORDER of a Differential Equation is the power of the highest derivative.

Q: What does it mean to find a SOLUTION of a differential equation?

A: A SOLUTION of a DE is a FUNCTION y(x) such that both sides of the DE are equal.

Ex 2. Verify that each function is a solution to the specified DIFFERENTIAL EQUATION.

A $y=12x^4$. DE: $xy'=4y$

Sol:

B $y=Ae^{\lambda x}$ DE: $y'=\lambda y$

Sol:

$\boxed{C}$ $y = Ce^{3t} - \frac{1}{3}$, D.E: $\frac{dy}{dt} = 1 + 3y$

Sol:

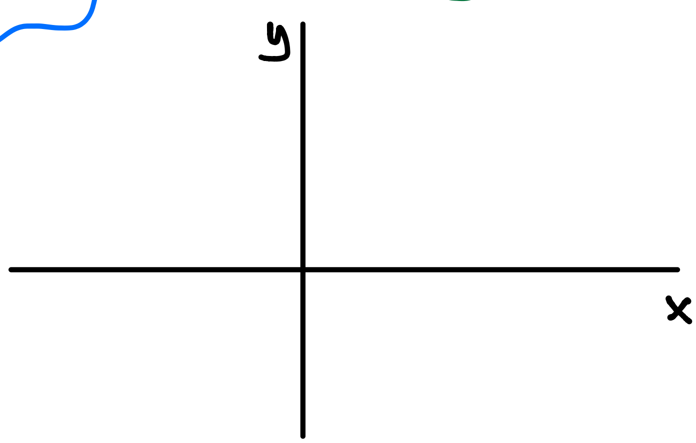
There are two different terms that are used for solutions to differential equations: PARTICULAR SOLUTIONS and the GENERAL SOLUTION

Ex. For the differential equation: $y' = 3x^2$

Find 3 PARTICULAR SOLUTIONS and then find the GENERAL SOLUTION

Sol: PART. SOL'NS:

GEN SOL'N:



* A common type of question in the field of differential equations is called an INITIAL VALUE PROBLEM (IVP). Here, you will be given a differential equation and an initial condition that the function (i.e. solution) needs to satisfy. The goal is to find a PARTICULAR SOLUTION that satisfies the differential equation AND the initial condition.

Ex 4. Solve the IVP: $y' = 3x^2, \quad y(0) = 4.$

Sol:

Ex 5: The general solution to a differential equation is given by: $y = Ce^{4x}$

Find the particular solution that satisfies the initial condition: $y(0) = 5$

Sol:

NOTE: If a particular solution has the form $y=K$ for some fixed constant K then we call this solution a **CONSTANT SOLUTION** (aka equilibrium solution). To find all constant solutions:

1. Let $y=K$ for a fixed constant K
2. Input $y=K$ into the DE (Note $y'=0$)
3. Solve for K (if possible)

Ex 6 Find all **CONSTANT SOLUTIONS** of each differential equation (if any exist)

$\boxed{A}$ $y'=y(y-2)$

sol:

$\boxed{B}$ $\frac{dy}{dt}=y^4-8y^3+12y^2$

sol:

$\boxed{C}$ $dy/dx=xy^2$ $\quad y=K$

sol:

$\boxed{D}$ $dP/dt=1+P\cdot t$ $\quad P=k$

sol:

PART 2: BEHAVIOR of SOLUTIONS

** Just by glancing at a differential equation, we can tell a lot about the BEHAVIOR of SOLUTIONS of the DE.

$dy/dx > 0 \Rightarrow$

$dy/dx < 0 \Rightarrow$

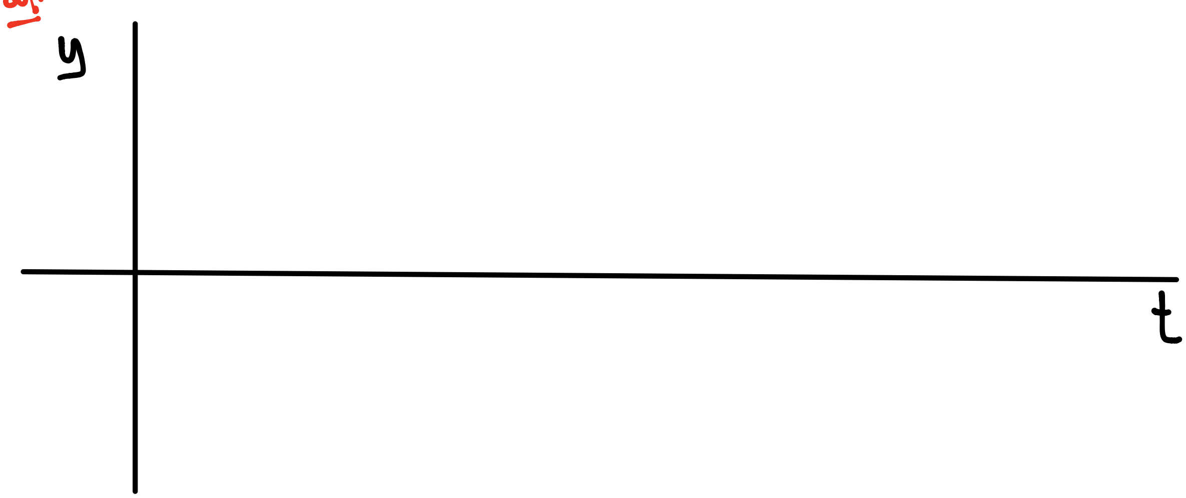
Ex 7. For each differential equation, sketch a graph that depicts the behavior of solutions to the DE.

$\boxed{A}$ $y' = y(y-2)$

sol:

$\boxed{B}$ $dy/dt = y^2 - 16 = (y+4)(y-4)$

sol:



PART 3: APPLICATIONS

** Differential equations can be used to <u>model</u> certain systems where a quantity changes over time. One of the most powerful applications comes when modeling <u>populations</u>

Ex 8. A population is modeled by the differential equation $\frac{dP}{dt} = 2P\left(1 - \frac{P}{100}\right)$

- For what values of P is the population INCREASING?
- For what values of P is the population DECREASING?
- Find all EQUILIBRIUM SOLUTIONS
- Sketch a plot illustrating this behavior.

LOGISTIC MODEL:

$$\boxed{\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)}$$

M = CARRYING CAPACITY
k = GROWTH RATE.

Sol: