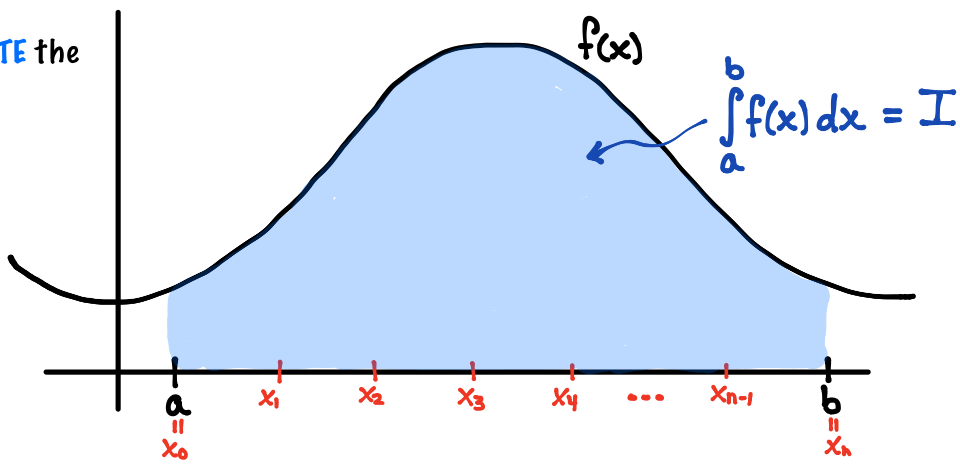
CH 7.7: APPROXIMATE INTEGRATION

MOTIVATION: The DEFINITE INTEGRAL $\int_a^b f(x)\,dx$ represents the signed area between the graph of the function f(x) and the x-axis (positive area above x-axis and negative below). Oftentimes, our integration methods DO NOT WORK, so we need to resort to APPROXIMATION TECHNIQUES.

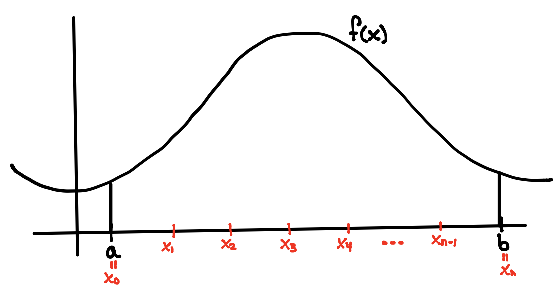
PART 1: THE 5 METHODS:

* We want to APPROXIMATE the blue area!

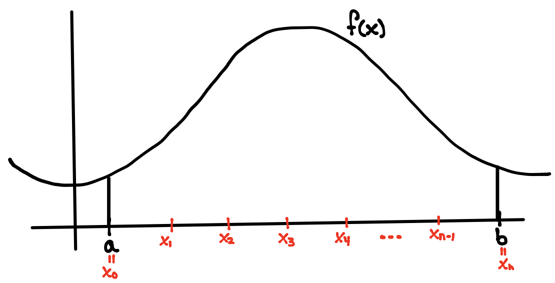


Number of intervals: n Width of each interval: $\Delta x = \frac{b-a}{n}$

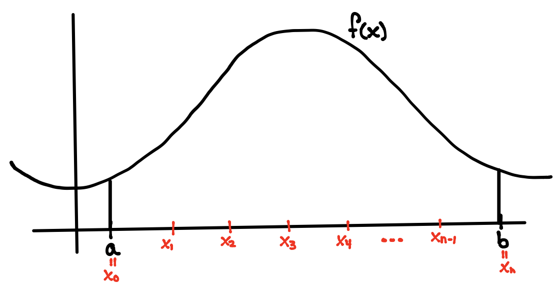
METHOD #1: [LEFT-HAND SUM] "L_n"



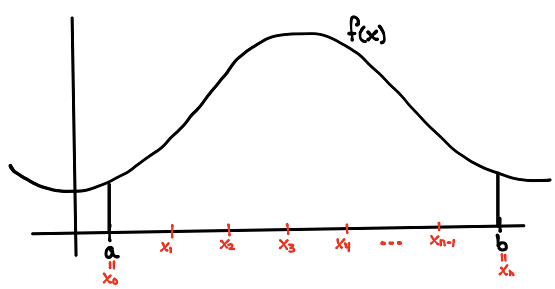
METHOD #2: [RIGHT-HAND SUM] "R_n"



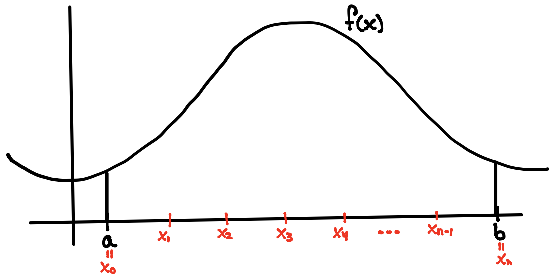
METHOD #3: [MIDPOINT RULE] "M_n"



METHOD #4: [TRAPEZOIDAL RULE] "T_n"



METHOD # 5: [SIMPSON'S RULE] "S_n"



PART 2: OVER or UNDER?

Rule	Overestimate of $\int_a^b f(x)dx$ when...	Underestimate of $\int_a^b f(x)dx$ when...
LEFT L_n		
RIGHT R_n		
TRAP T_n		
MID M_n		

PART 3: ERROR IN APPROXIMATIONS of $\int_a^b f(x)\,dx$

$$E_T = \int_a^b f(x)\,dx - T_n$$

$$E_M = \int_a^b f(x)\,dx - M_n$$

$$E_S = \int_a^b f(x)\,dx - S_n$$

THM: [ERROR BOUND for TRAP and MID]

Suppose $|f''(x)| \le K$ for $a \le x \le b$. If E_T and E_M are the errors in the TRAPEZOIDAL RULE and MIDPOINT RULE, then:

THM: [ERROR BOUND for SIMPSON'S RULE]

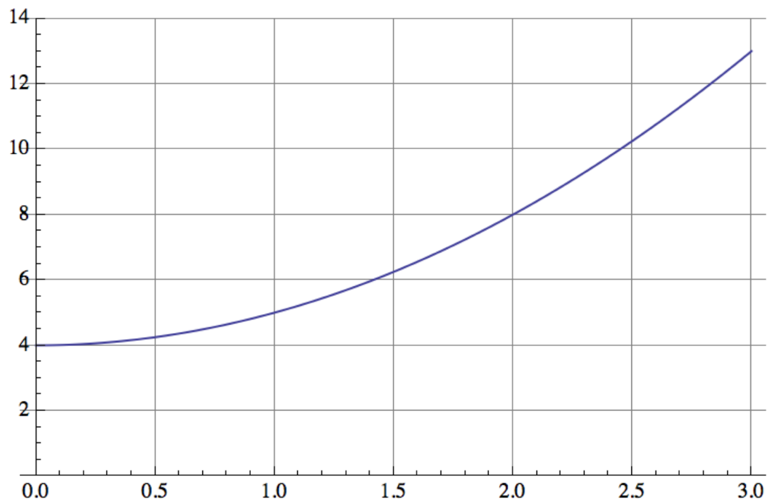
Suppose $|f^{(4)}(x)| \le K$ for $a \le x \le b$. If E_S is the error involved in using SIMPSON'S RULE, then:

NOTE:

- ◻
- ◻
- ◻
- ◻
- ◻

PART 4: EXAMPLES

Ex 1. Given the graph of f(x), let $I = \int_a^b f(x)\,dx$ and find the following approximations of I. Also label each as an over or under estimate:



A L_3

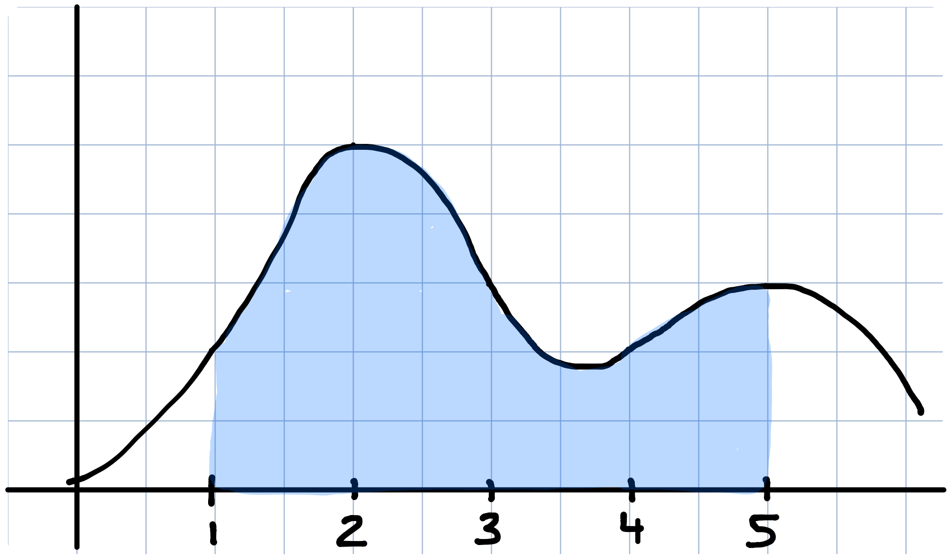
B R_3

C M_3

D T_3

List I, R_3, L_3, M_3, T_3 in order from least to greatest:

Ex 2: Use the TRAPEZOIDAL RULE, SIMPSON'S RULE, and MIDPOINT RULE with $n=4$ to approximate the area under the graph shown below:



sol:

Ex 3. Estimate the value of the definite integral $\int_1^2 \frac{1}{x}\,dx$ using $n=5$ and the following approximation methods:

A L_4

B R_4

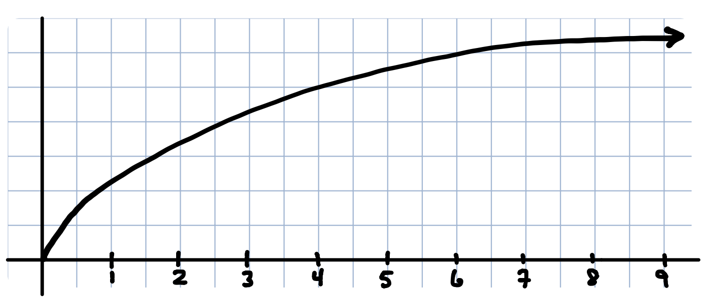
C M_4

D T_4

E S_4

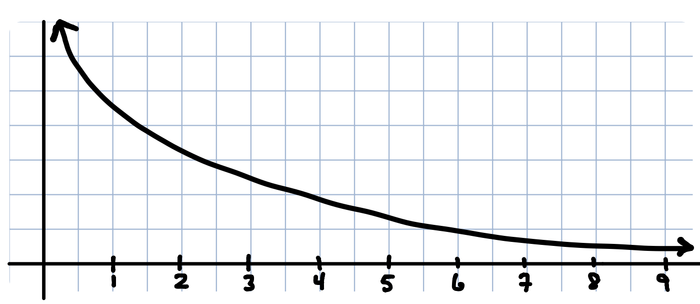
Ex 4. For each f(x) drawn below, list $\int_1^9 f(x)\,dx$, T_8, M_8, R_8, L_8 in order from smallest to largest.

A



sol:

B



sol:

Ex 5. How large should we take "n" in order to guarantee that the **TRAPEZOIDAL RULE** and **MIDPOINT RULE** approximations for $\int_1^2 1/x \, dx$ are accurate to within 0.0001?

sol:

Ex 6. How large should "n" be to guarantee that the **SIMPSON'S RULE** approximation of $\int_0^1 4e^{x^2} dx$ is accurate to within 0.0001?

sol: