CH 7.4 PARTIAL FRACTIONS METHOD of INTEGRATION.

MOTIVATION: Compute the following indefinite integrals:

A $\int \frac{4}{x} + \frac{2}{x+1}\,dx$

sol:

B $\int \frac{6x+4}{x^2+x}\,dx$

sol:

GOAL: We want to be able to take an expression

and rewrite it as

in order to more easily integrate! This is called the METHOD of PARTIAL FRACTION DECOMPOSITION

REVIEW: Some things you ought to master before we begin:

① LOG INTEGRALS: $\int \frac{1}{x}\,dx = \ln|x| + C$

Ex. A $\int \frac{1}{x+4}\,dx$

B $\int \frac{1}{4-x}\,dx$

C $\int \frac{1}{2x+1}\,dx$

D $\int \frac{1}{5-3x}\,dx$

② FACTORING: ALWAYS check your factoring (by expanding it back out) before you proceed with the integration!

③ ARCTAN INTEGRALS: $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$ for $a \neq 0$

Ex. A $\int \frac{1}{x^2+25}\,dx$

B $\int \frac{6}{4x^2+9}\,dx$

PART 1: THE PROCEDURE

* We will consider integrals involving RATIONAL EXPRESSIONS of the form $P(x)/Q(x)$, where $P(x)$ and $Q(x)$ are Polynomials (with the degree of $P(x)$ less than the degree of $Q(x)$)

STEP 1: FACTOR the denominator fully

STEP 2: Determine the FORM of the PARTIAL FRACTION decomposition depending on the types of factors in denominator. We will deal with two cases:

- Distinct LINEAR Factors:

- Distinct, irreducible QUADRATIC Factors:

STEP 3: Solve for the UNKNOWN PARAMETERS. Multiply both sides of equation by the denominator, then make judicious choices for x (see example!)

STEP 4: With the new simplified form, you can proceed to INTEGRATE. If all went accordingly you should be able to apply substitution or use arctan rule.

PART 2: SOME EXAMPLES

Ex 1. Compute the following integrals using PARTIAL FRACTION DECOMPOSITION.

A $\int \frac{6x+4}{x^2+x}\,dx$

Sol:

B $\int \frac{11x-2}{x^2-x-12}\,dx$

Sol:

C $\int \frac{2x^2-3x+8}{x^3+4x}\,dx$

Sol:

D $\int \frac{2x^2+17x-12}{x^3+5x^2-6x}\,dx$

Sol:

E $\int \frac{2}{x^3-9x}\,dx$

Sol:

NOTE: We can treat cases where the degree of P(x) is greater than or equal to the degree of Q(x) by using polynomial long division. This is beyond the scope of the course. Repeated factors can also be treated, but we will not cover that here.

PART 3: SMARTER NOT HARDER

Ex 2. Compute the following integrals:

A $\int \frac{dx}{x^2+4x+7}$

Sol:

B $\int \frac{x+2}{x^2+2x+5}\,dx$

sol: