CH 11.9 POWER SERIES REPRESENTATION of FUNCTIONS

GOAL: We are going to learn about an important application of power series. Using what we know about geometric series, we can represent certain functions (on particular intervals) as power series!

PART 1: FINDING POWER SERIES REPRESENTATIONS

GEO SERIES: $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots$ $\begin{cases} \text{CONVERGES TO } \frac{a}{1-r} \text{ if } |r|<1 \\ \text{DIVERGES if } |r| \geq 1 \end{cases}$

** Replace r with the variable x and let a=1 and we get the following:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \quad \text{PROVIDED } |x|<1 \quad (\text{i.e } x \in (-1,1))$$

POWER SERIES REPRESENTATION of $f(x) = \frac{1}{1-x}$

WHY DO WE CARE?:

** We can use this to find POWER SERIES REPRESENTATIONS and corresponding intervals of convergence for other functions that have a "similar" form.

Ex 1. Find the POWER SERIES REPRESENTATION of each of the following functions and indicate the interval of convergence.

A $f(x) = \frac{1}{1-2x}$

sol:

$\boxed{B}$ $f(x) = \frac{1}{1+x^2}$

sol:

$\boxed{C}$ $f(x) = \frac{4}{x+5}$

sol:

$\boxed{D}$ $f(x) = \frac{6}{x^2-2x-8}$ **HINT:** PARTIAL FRACTIONS.

sol:

E $f(x) = \frac{x}{1-x^5}$

Sol:

PART 2: CALCULUS w/ POWER SERIES

RECALL:

$$\left.\begin{aligned} \frac{d}{dx}\left[f_1(x) \pm f_2(x)\right] &= \frac{d}{dx}\left[f_1(x)\right] \pm \frac{d}{dx}\left[f_2(x)\right] \\ \int \left[f_1(x) \pm f_2(x)\right] dx &= \int f_1(x)\,dx \pm \int f_2(x)\,dx \end{aligned}\right\}$$

TERM-BY-TERM DIFFERENTIATION/ INTEGRATION.

* This property of derivatives and Integrals still holds for an infinite sum!

THM: [DERIVATIVES/INTEGRALS of POWER SERIES]

If the power series $\sum_{n=0}^{\infty} C_n(x-a)^n$ has radius of convergence $R>0$, then the function:

is differentiable (and hence continuous) on the interval $(a-R, a+R)$ and:

- The DERIVATIVE is
- The INTEGRAL is

NOTE:

A SHORTCUT

w/ SUMMATION NOTATION

Ex 2: Consider the function $f(x) = \frac{1}{(1-x)^2}$

A Use differentiation to find a POWER SERIES REPRESENTATION for f(x). What is the radius of convergence of this new power series?

sol:

B Use part [A] to find a POWER SERIES REPRESENTATION for $f(x) = \frac{1}{(1-x)^3}$ What is the new radius of convergence?

sol:

$\boxed{C}$ Use part [B] to find a POWER SERIES REPRESENTATION for $f(x) = \dfrac{x}{(1-x)^3}$
What is the new radius of convergence?

sol:

Ex 3. Evaluate the integral as a power series and indicate the radius of convergence.

$$\int \frac{x}{1-x^5}\,dx$$

sol: