CH 11.8: POWER SERIES

THE MAIN IDEA:

What numbers can we input into this series to have the result be a CONVERGENT series?

$$\sum_{n=0}^{\infty} \frac{(\square - 1)^n}{2^n}$$

* Typically, the green box is replaced by a variable "x" and we create what is called a POWER SERIES.

DEFN: [POWER SERIES]

A POWER SERIES centered at "a" is a series with the form:

$$\sum_{n=0}^{\infty} C_n(x-a)^n = C_0 + C_1(x-a)^1 + C_2(x-a)^2 + C_3(x-a)^3 + \dots$$

Where "x" is a variable and the C_n's are constants that we refer to as the coefficients of the series.

NOTE:

GOAL: Determine the values of "x" for which $\sum_{n=0}^{\infty} C_n(x-a)^n$ will converge.

Ex 1. Find all values of x for which the following power series converges:

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n}$$

sol:

There are three different possibilities for a given power series: $\sum_{n=0}^{\infty} C_n (x-a)^n$

- The series converges for all values of x. In this case:

RADIUS of CONVERGENCE

INTERVAL OF CONVERGENCE

- The series converges only when x=a. In this case:

RADIUS of CONVERGENCE

INTERVAL OF CONVERGENCE

- There exists some value R>0 (called the radius of convergence) such that the series converges for $|x-a| < R$. In this case:

RADIUS of CONVERGENCE

INTERVAL OF CONVERGENCE

THE PROCEDURE

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RECALL

THM: [THE RATIO TEST]

$\sum a_n$. $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$

$L<1$ ($L=0$ is o.k.)	$L>1$ ($L=\infty$ is o.k.)	$L=1$

Ex 2. Find the RADIUS of CONVERGENCE and INTERVAL of CONVERGENCE for each of the following power series:

A $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

sol:

B $\sum_{n=1}^{\infty} n!\,x^n$

sol:

C $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (x-1)^n}{n}$

sol:

D $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{3^n \cdot n^2}$

sol:

Ex 3. (i) Find the RADIUS of CONVERGENCE and INTERVAL of CONVERGENCE for each of the following power series: $\sum_{n=1}^{\infty} \frac{2^{n+1}}{\sqrt{n}} \cdot x^n$

sol:

(ii) Knowing what we know from part (i), what can we tell about the following series:

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{\sqrt{n}} \cdot x^{2n}$$

Sol:

Ex 4. Suppose that the **RADIUS of CONVERGENCE** of the power series $\sum_{n=0}^{\infty} C_n X^n$ is R. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} C_n X^{3n}$

Sol:

Ex 5: Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x=-2$ and diverges when $x=4$. What can be said about the CONVERGENCE/DIVERGENCE of the following series?

Sol: