CH 11.6 ABSOLUTE CONVERGENCE and RATIO TEST

GOAL: We will discuss a property of convergence that some series possess and we will learn our <u>final</u> convergence test of this unit!

PART 1: ABSOLUTE and CONDITIONAL CONVERGENCE

* Given a series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \dots$ ← SOME TERMS MAY BE NEGATIVE

we consider the corresponding series

$$\sum_{n=1}^{\infty} |a_n| = |a_1| + |a_2| + |a_3| + |a_4| + \dots$$

DEFN: A series $\sum a_n$ is:

- ABSOLUTELY CONVERGENT if:

- CONDITIONALLY CONVERGENT if:

- DIVERGENT if:

(!) Note that if the series has all positive terms then $|a_n| = a_n$ so absolute convergence is the same as convergence in this case.

Ex 1. Determine if the following series are ABSOLUTELY CONVERGENT, CONDITIONALLY CONVERGENT, or DIVERGENT.

A $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

Sol:

B $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

Sol:

C $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{\sqrt{n^3+1}}$

sol:

D $\sum_{n=1}^{\infty} \frac{\cos(2n)}{2^n}$

sol:

* There is an important relationship between the convergence of $\sum a_n$ and $\sum |a_n|$

THM:

If a series $\sum a_n$ is **ABSOLUTELY CONVERGENT** then it is CONVERGENT.

(I.e. If $\sum |a_n|$ converges then $\sum a_n$ converges too!)

NOTE

PART 2: THE RATIO TEST

* Currently, we only have one test for convergence that applies to series with positive and negative terms. We will now learn a second one, called the RATIO TEST! This test is very useful in determining if a series is absolutely convergent.

THM: [THE RATIO TEST]

Given the series $\sum a_n$. if:

Ex 2. Use the RATIO TEST to determine if each series is ABSOLUTELY CONVERGENT, CONDITIONALLY CONVERGENT, or DIVERGENT

A $\sum_{n=1}^{\infty} \frac{6^n}{n^2}$

Sol:

B $\sum_{n=1}^{\infty} \frac{(-2)^n}{n \cdot 3^n}$

Sol:

The **RATIO TEST** works great with series involving factorials! This means that you will need to be comfortable working with factorials.

C $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

sol:

D $\sum_{n=1}^{\infty} \frac{n^2}{(2n)!}$

sol:

E $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

sol:

! The RATIO TEST is inconclusive for P-Series since $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$