TESTS for CONVERGENCE

$$\sum_{n=1}^{\infty} a_n$$

$\lim_{n\to\infty} a_n = 0$

w/ form $\sum ar^n$

GEOMETRIC SERIES TEST

- CONVERGES if $|r| < 1$
- DIVERGES if $|r| \geq 1$

$a(n) = f(n)$. IF $f(n)$ $\oplus$ & $\downarrow$

INTEGRAL TEST

- CONVERGES OR DIVERGES WHEN $\int f(n)\,dn$ DOES.

$\lim_{n\to\infty} a_n \neq 0$

DIVERGENCE TEST

$\sum a_n$ DIVERGES

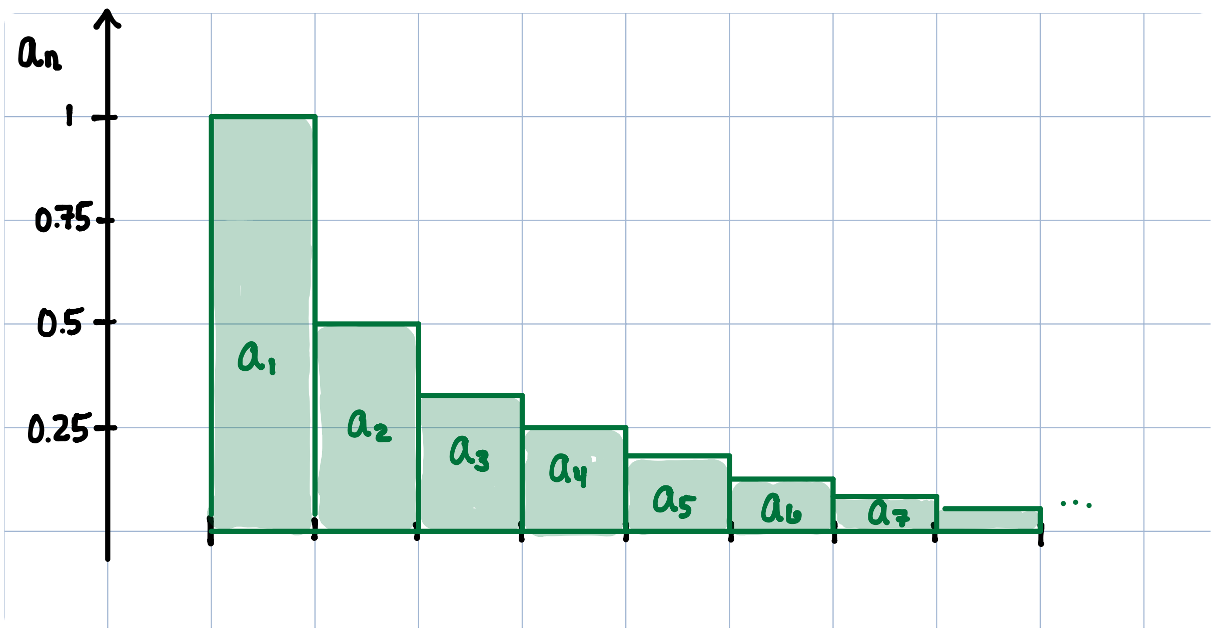
CH 11.3: THE INTEGRAL TEST

GOAL: We are going to learn another test that can be performed on certain infinite series to determine if they CONVERGE (i.e. have a fixed, finite sum) or DIVERGE (i.e. do not). This test is called the INTEGRAL TEST and it involves comparing an infinite series to an improper integral.

PART 1: VISUALIZING A SERIES

* To be able to fully understand the integral test, we first need a way to visualize a series.

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots$$



To find if a series converges, we need to see if this area is a fixed, finite value! We will use what we know about improper integrals.

PART 2: THE MAIN IDEA

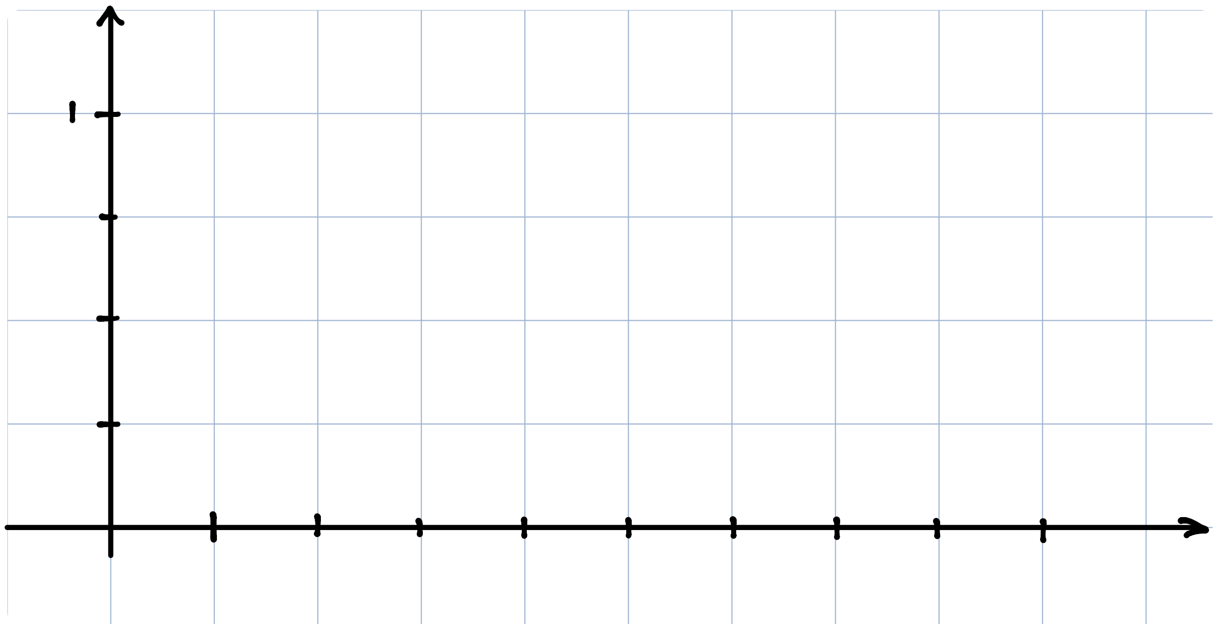
* To illustrate the idea if the INTEGRAL TEST, we will consider two separate examples.

A $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES

B $\sum_{n=1}^{\infty} \frac{1}{n^2}$ CONVERGES

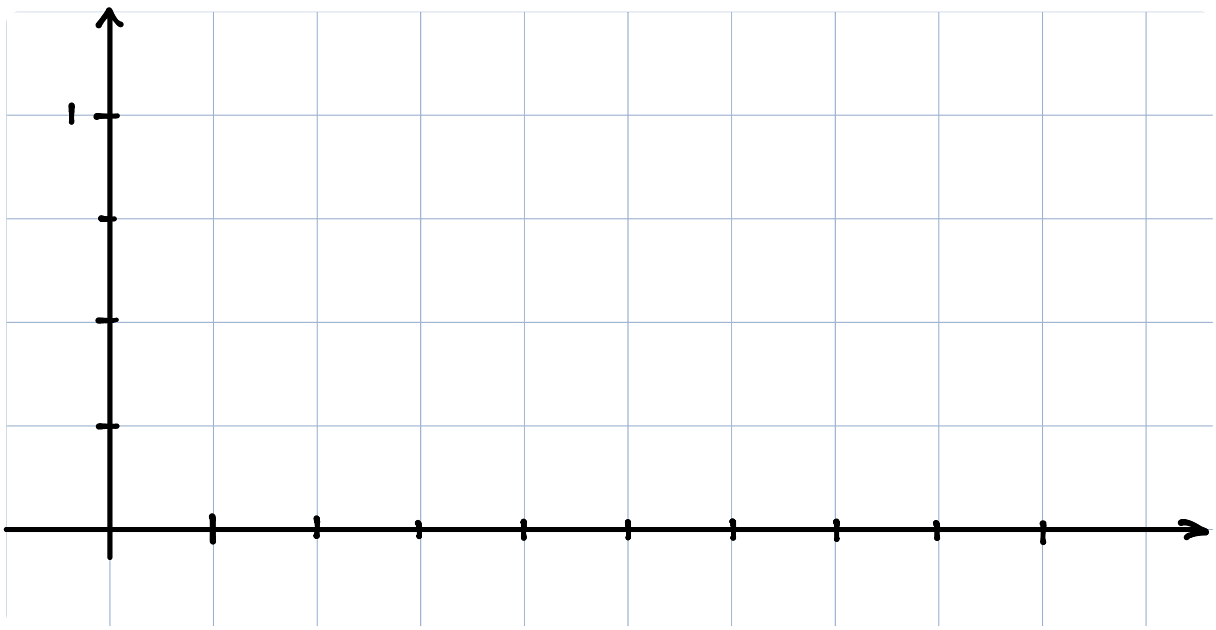
$\boxed{A}$ $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

sol:



$\boxed{B}$ $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + 1/4 + 1/9 + 1/16 + 1/25 + \cdots$

Sol:



PART 3: THE INTEGRAL TEST

THM: [THE INTEGRAL TEST]

TIPS for showing a fn is **DECREASING**

Ex 1: Test each series for CONVERGENCE or DIVERGENCE.

A $\sum_{n=1}^{\infty} \frac{2n}{n^2+2}$

sol:

B $\sum_{n=1}^{\infty} 4ne^{-2n^2}$

sol:

C $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

Sol:

Ex2. THE P-TEST FOR SERIES

RECALL:

THM: [P-TEST TYPE 1]

$$\int_a^{\infty} \frac{1}{x^p}\,dx \quad \begin{cases} \text{CONVERGES if } p>1 \\ \text{DIVERGES if } p\le 1 \end{cases}$$

for $a>0$

Determine values of "p" for which the series **CONVERGES** and for which the series **DIVERGES.**

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

Sol:

THM: [P-TEST FOR SERIES]

A $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

B $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

PART 4: ESTIMATING THE SUM of A SERIES.

* Using the integral test, we can determine that a given series converges but we CANNOT determine what it converges to. We can, however, get a good approximation by using partial sums. Naturally, approximations are not exact. We define the REMAINDER to be the difference between the exact value of a sum and the approximate value

RECALL:

$S_n = n^{th}$ PARTIAL SUM

Sum of SERIES $= S = \lim_{n\to\infty} S_n$

REMAINDER: $\boxed{R_n = S - S_n}$

NOTE:

* Using ideas similar to the integral test, we can figure out approximately how big the remainder will be... These are called REMAINDER ESTIMATES.

THM: [REMAINDER ESTIMATES]

Ex 3. Consider the series $\sum_{n=1}^{\infty} 4ne^{-2n^2}$

A Find an explicit upper bound for the remainder R_n when estimating the series with the nth partial sum.

Sol:

$\boxed{B}$ Find an n for which the upper bound on R_n is less than 0.0001, and then compute the nth partial sum to 5 digits for this specific n.

sol: