CH 11.1: SEQUENCES

PART 1: THE BASICS

DEFN: [SEQUENCE]:

NOTATION

There are 3 ways to express a SEQUENCE

Ex 1. [FINDING TERMS of A SEQUENCE]

For each of the following sequences, give the formula for the nth term of the sequence and write out the first several terms of the sequence.

A $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$

B $\left\{(-1)^n \cdot n^2\right\}_{n=0}^{\infty}$

C $\left\{\sqrt{n-2}\right\}_{n=2}^{\infty}$

D $\left\{\sin(n\pi)\right\}_{n=1}^{\infty}$

Ex 2. [FINDING a_n]

For each of the following sequences, give a formula for the general term a_n of the sequence, assuming that the given pattern continues.

A $\{2, 4, 8, 16, 32, 64 \ldots\}$

Sol:

! HOW ABOUT $\{1, 2, 4, 8, 16, \ldots\}$

$\boxed{B}$ $\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots\}$

Sol:

$\boxed{C}$ $\{\frac{4}{3}, \frac{5}{9}, \frac{6}{27}, \frac{7}{81}, \dots\}$

Sol:

$\boxed{D}$ $\{-\frac{7}{2}, \frac{7}{5}, -\frac{7}{8}, \frac{7}{11}, -\frac{1}{2}, \dots\}$

Sol:

NOTE: The formulae found above are referred to as the CLOSED FORM of the respective sequences. Not all sequences have a closed form.

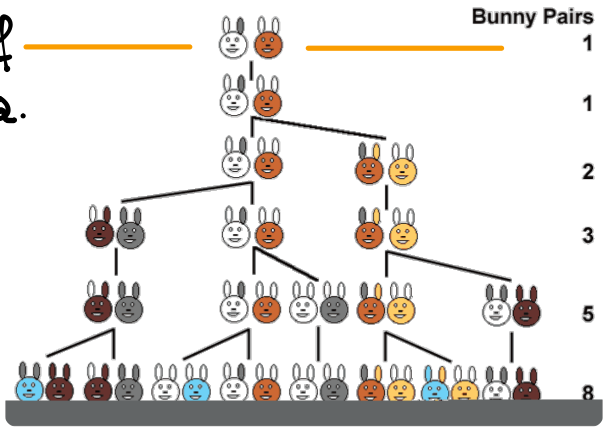
Ex. $\{7,1,8,2,8,1,8,2,8,4,5,\dots\}$

** Some sequences can be defined by relating each term to the preceding terms of the sequence. This is called a RECURSIVE SEQUENCE.

DEFN: [RECURSIVE SEQUENCE]

Ex.

THE ORIGIN of FIBONACCI SEQ.



Ex 3. [FINDING TERMS of A RECURSIVE SEQ]. Find the first 4 terms of each sequence:

A $a_n = 3a_{n-1} + 2$ for $n \geq 2$
$a_1 = 1$

Sol:

B $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$
$a_1 = 1$ $a_2 = 2$

Sol:

Ex 4. [FINDING RECURSIVE FORMULA] Find a recursive formula for each sequence:

A $\{1, 2, 6, 24, 120, \ldots\}$

Sol:

B $\{1, 3, 7, 15, 31, \ldots\}$

Sol:

PART 2: HOW TO VISUALIZE A SEQUENCE

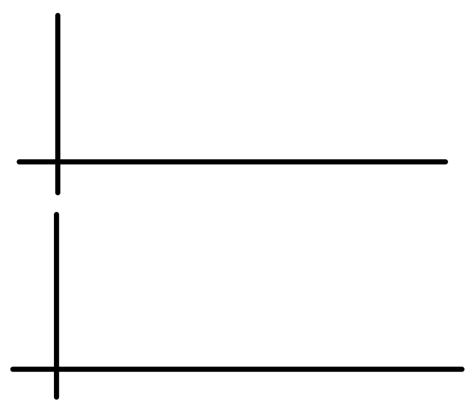
* Plot n on x-axis and a_n on y-axis.

> We could also just plot the terms on a number line, but this is usually not very helpful.

PART 3: THE LIMIT of A SEQUENCE.

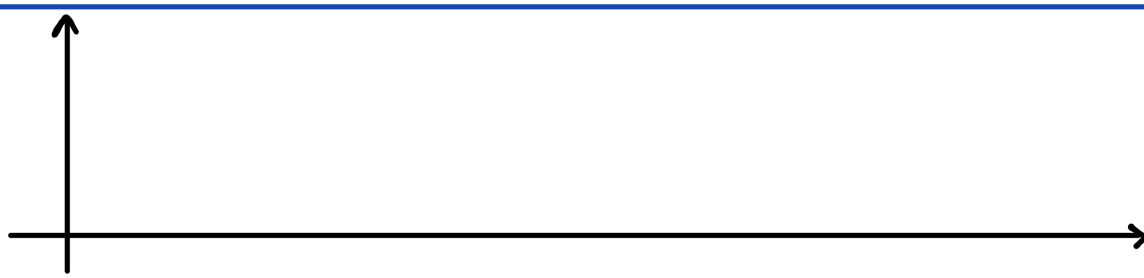
* We often want to know what happens to the terms of a sequence as n goes to infinity. This is called the **LIMIT OF THE SEQUENCE** and is denoted by:

DEFN: [LIMIT of A SEQUENCE]



NOTE: A <u>more precise</u> definition of this is given next:

DEFN: [LIMIT of A SEQUENCE]



To compute limits of sequences, all of the rules that we had for functions still apply. That is:

PROPERTIES of LIMITS

RULES for LIMITS

Ex 5. [CONVERGENCE of SEQUENCES].

Determine if the following sequences CONVERGE or DIVERGE. If it converges, find the limit.

$\boxed{A}$ $\left\{\left(\frac{2}{3}\right)^n\right\}_{n=0}^{\infty}$

sol:

$\boxed{B}$ $\left\{\left(\frac{3}{2}\right)^n\right\}_{n=0}^{\infty}$

sol:

$\boxed{C}$ $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

sol:

$\boxed{D}$ $\left\{\frac{5+2n^2}{4+8n^2}\right\}_{n=1}^{\infty}$

sol:

$\boxed{E}$ $\left\{\frac{n^2+1}{\sqrt{n^3+4}}\right\}_{n=0}^{\infty}$

sol:

$\boxed{F}$ $\left\{\frac{e^n+1}{e^n-1}\right\}_{n=1}^{\infty}$

sol:

THM: [LIMITS IN CONTINUOUS fn's]

G $\left\{ e^{4/n} \right\}_{n=1}^{\infty}$

sol:

H $\left\{ \ln\left(\frac{n^2+1}{n^2+6}\right) \right\}_{n=1}^{\infty}$

sol:

! Limits of **ALTERNATING SEQUENCES** can be tricky, as evidenced by the next few examples.

I $\left\{ \cos(n\pi) \right\}_{n=1}^{\infty}$

sol:

J $\left\{ \frac{(-1)^n}{n} \right\}_{n=1}^{\infty}$

sol:

PART 4: SOME CHARACTERISTICS of SEQUENCES