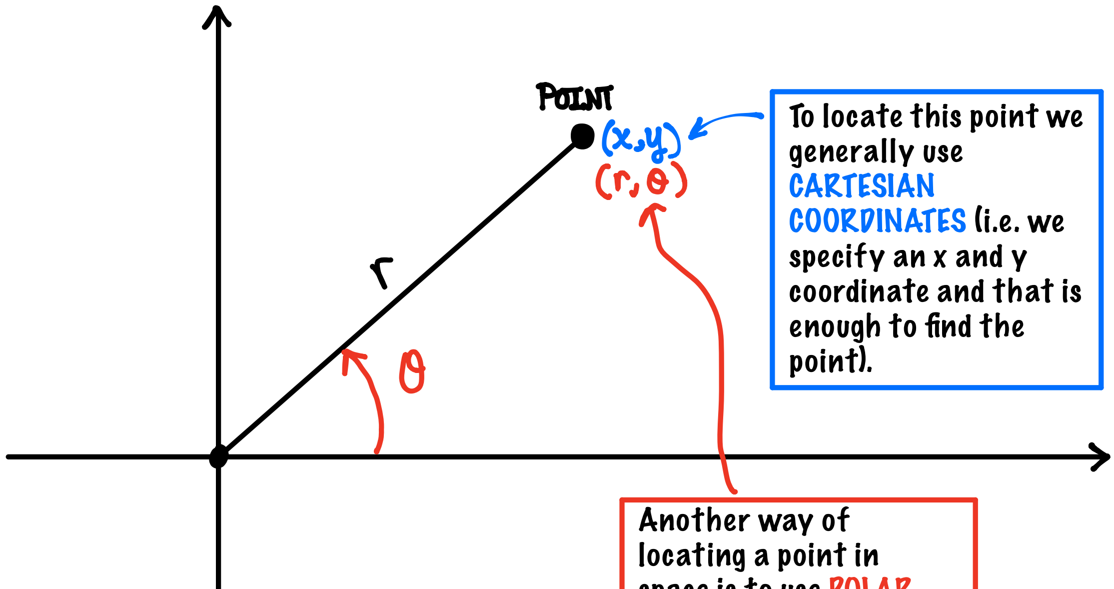
CH 10.3 POLAR COORDINATES

PART 1: AN INTRODUCTION

** In 2D we use the XY Cartesian Plane to help us organize points in space:



To locate this point we generally use CARTESIAN COORDINATES (i.e. we specify an x and y coordinate and that is enough to find the point).

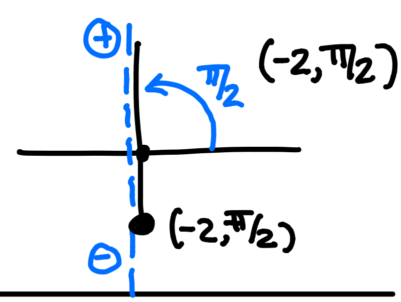
Another way of locating a point in space is to use POLAR COORDINATES where we specify an angle and a radius.

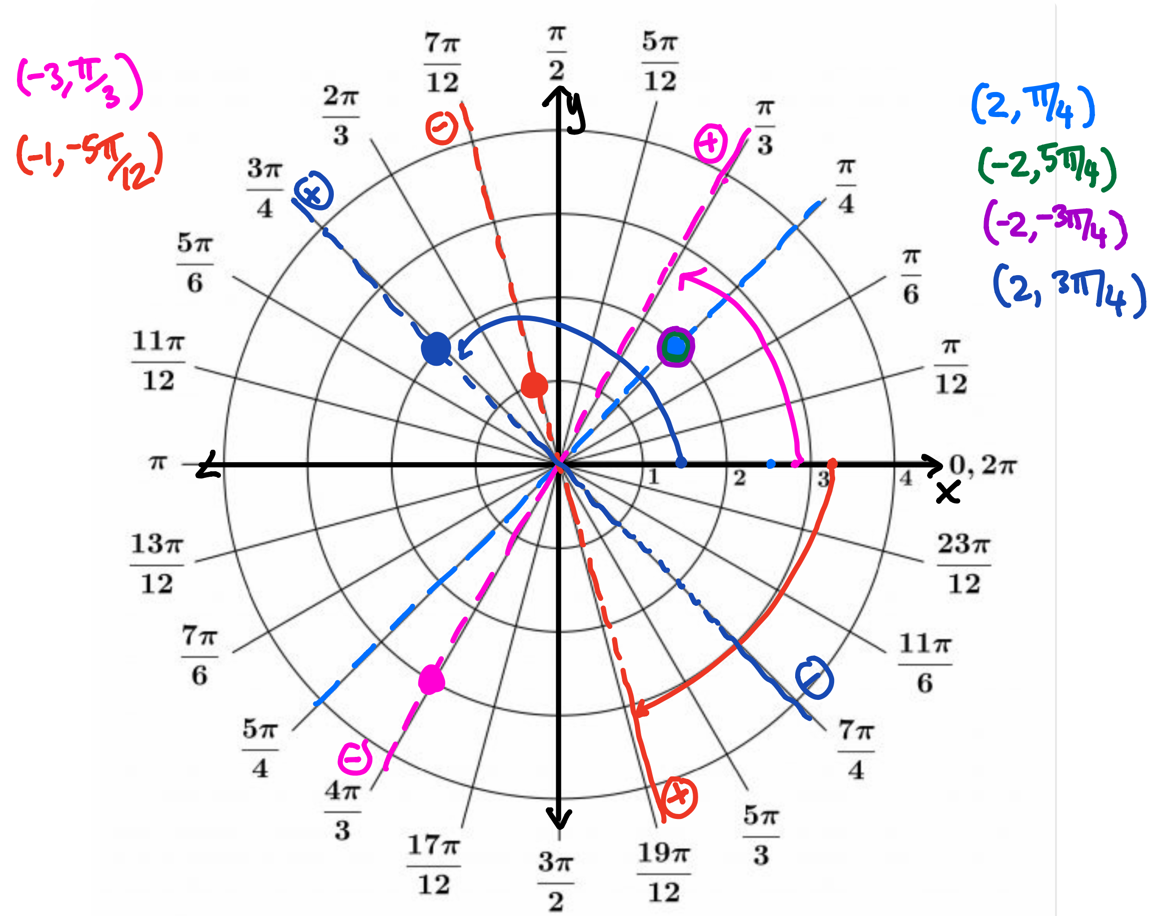
NOTE: Polar coordinates NOT unique. $(4, \pi/4)$. SAME $(4, 9\pi/4)$ or $(4, -7\pi/4)$

Some TRICKY things about polar coordinates:

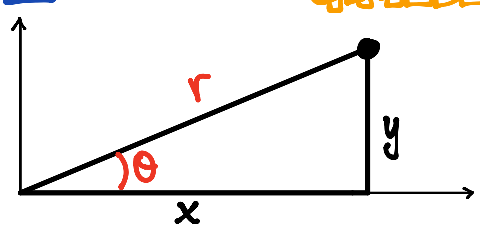
NEGATIVE "θ"

NEGATIVE "r" (Plot in opp direction)





PART 2: CONVERTING CARTESIAN ⇔ POLAR



POLAR ⇒ CARTESIAN

$(r, \theta) \Rightarrow$ $x = r\cos\theta$
$y = r\sin\theta$.

$\sin\theta = y/r \Rightarrow y = r\sin\theta$

$\cos\theta = x/r \Rightarrow x = r\cos\theta$

$r^2 = x^2 + y^2$

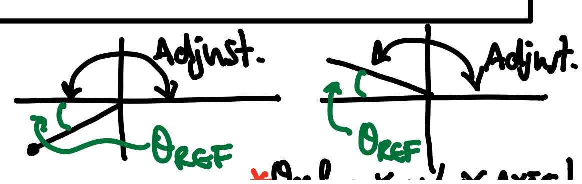
$r = \sqrt{x^2+y^2}$

$\tan\theta = y/x \Rightarrow \boxed{\theta = \arctan(y/x)}$

CARTESIAN ⇒ POLAR

$(x, y) \Rightarrow$ $r = \sqrt{x^2+y^2}$

$\theta_{REF} = \arctan(|y/x|)$ ADJUST to find θ.



*θ_{ref} = ∡ w/ x axis :

Ex 1. Convert each order pair from Cartesian coordinates to POLAR COORDINATES.

> * Can you find another polar coordinate for this ordered pair? How about with negative "r"?

A (3,4)

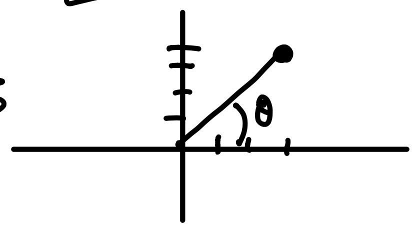
Sol: $x=3$
$y=4$

$r = \boxed{5}$

$\theta = \boxed{.927}$

$(5, 0.927)$

$r=\sqrt{3^2+4^2}=5$



$\theta = \arctan(4/3) = 0.927$ RADS

B (-4,-4)

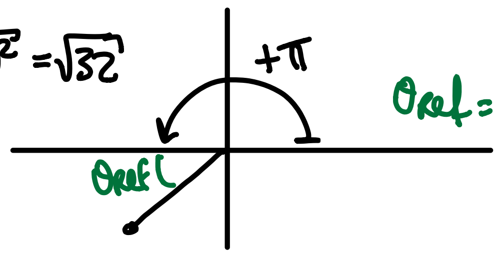
Sol: $x=-4$
$y=-4$

$r=\boxed{\sqrt{32}}$

$\theta = \boxed{5\pi/4} = \theta_{REF} + \pi = \pi/4 + \pi.$

$(\sqrt{32}, 5\pi/4)$

$r=\sqrt{(-4)^2+(-4)^2}=\sqrt{32}$



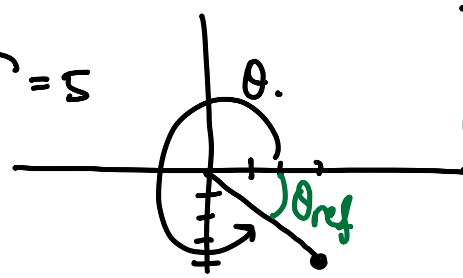
$\theta_{ref} = \arctan(|y/x|) = \arctan(1)$
$= \pi/4$

C (3,-4)

Sol: $x=3$
$y=-4$

$r=\boxed{5}$ $\quad \theta = \boxed{5.355} = \underline{2\pi - \theta_{ref}}$
Adjustment.

$r=\sqrt{3^2+(-4)^2}=5$



$\theta_{ref} = \arctan(|y/x|)$
$= \arctan(4/3)$
$= 0.927$

Ex 2. Convert each coordinate from polar coordinates to CARTESIAN COORDINATES

A $(\overset{r}{2}, \overset{\theta}{\pi})$

Sol: $x = r\cos\theta = 2\cos(\pi) = -2$
$y = r\sin\theta = 2\sin(\pi) = 0$

$(-2, 0)$

B $(42, -\pi/2)$

Sol: $x = 42\cos(-\pi/2) = 0$
$y = 42\sin(-\pi/2) = -42$

$(0, -42)$

PART 3: POLAR CURVES

$$\boxed{r = f(\theta)}$$

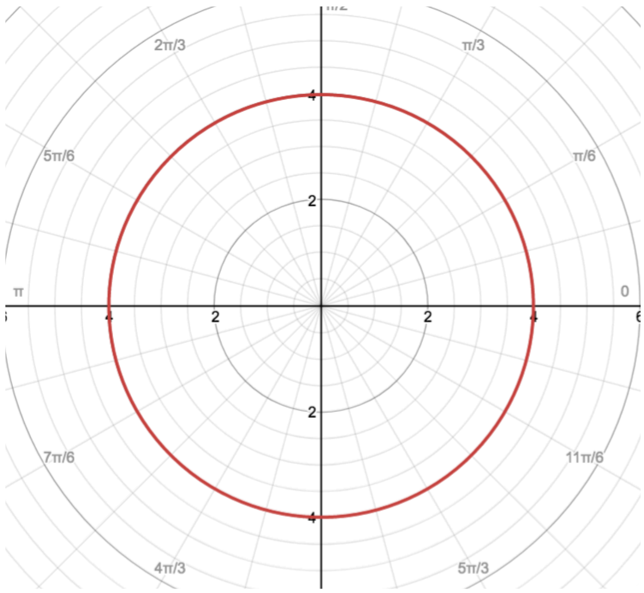
* Given a function in **POLAR COORDINATES**, we can sketch a curve by creating a table of values (for various angles, find the corresponding radius)

Ex 3. Graph the following:

A $r = 4$

sol:

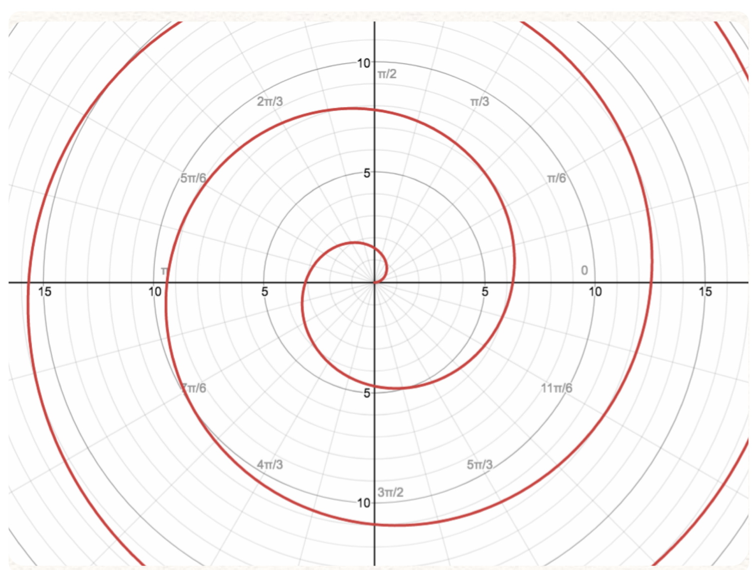
θ	r
0	4
$\pi/4$	4
$\pi/2$	4
$3\pi/4$	4
π	4
$\vdots$	



B $r = \theta$

sol:

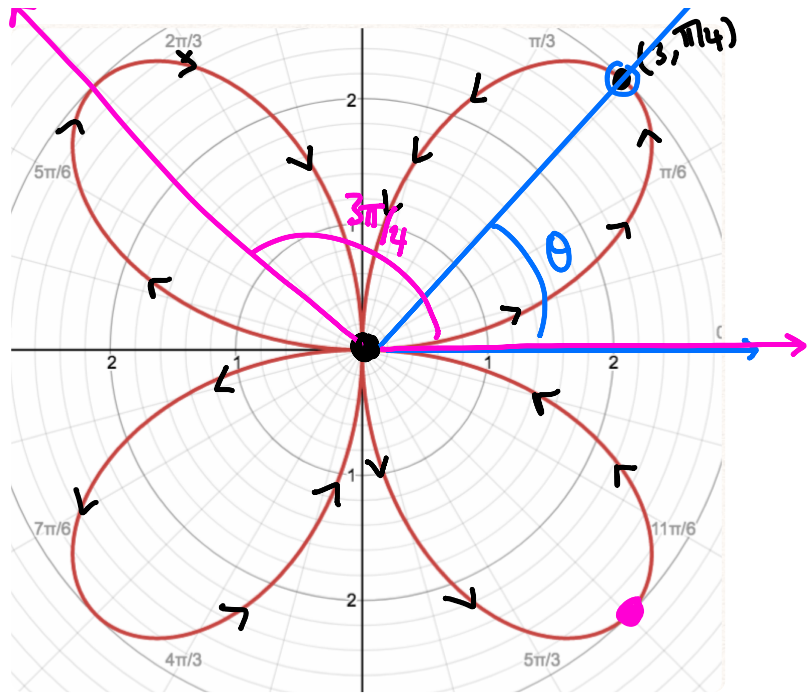
θ	r
0	0
$\pi/4$	$\pi/4$
$\pi/2$	$\pi/2$
$3\pi/4$	$3\pi/4$
π	π.
$\vdots$	



C $r = 3\sin(2\theta)$

sol:

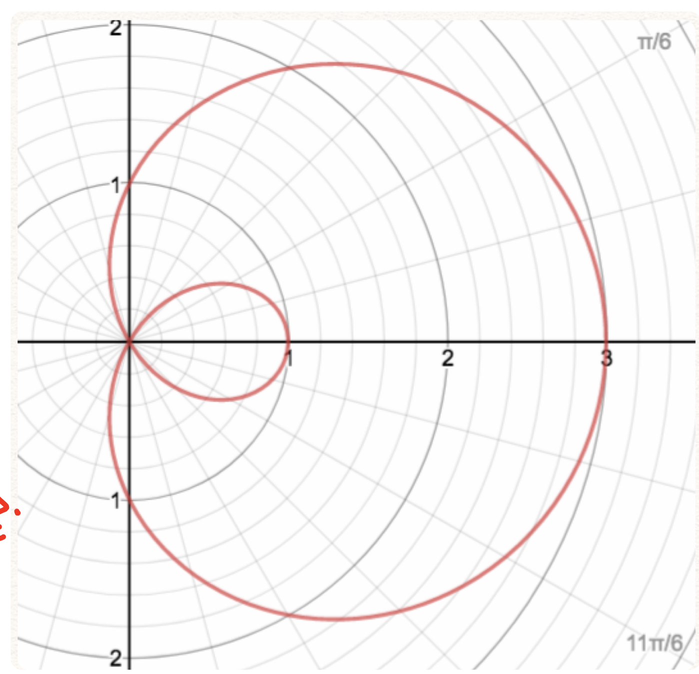
θ	r
0	0
$\pi/4$	3
$\pi/2$	0
$3\pi/4$	-3
π	
$\vdots$	



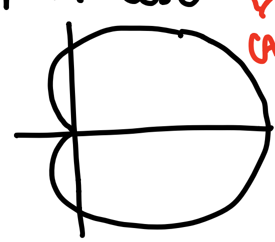
D $r = 1 + 2\cos(\theta)$

sol:

θ	r
0	
$\pi/4$	
$\pi/2$	
$3\pi/4$	
π	
$\vdots$	



$r = 1 - \cos\theta$ ♡ CARDIOID.

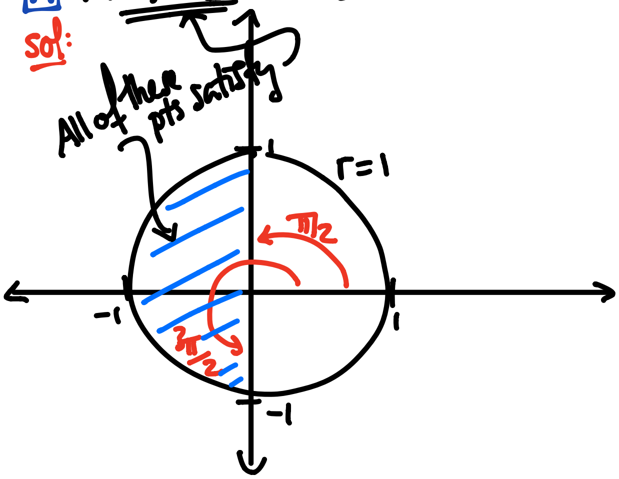


NOTE: We can also easily sketch regions (wedges, discs, etc) using polar coordinates and inequalities. Check it out:

Ex 4: Sketch each of the following regions:

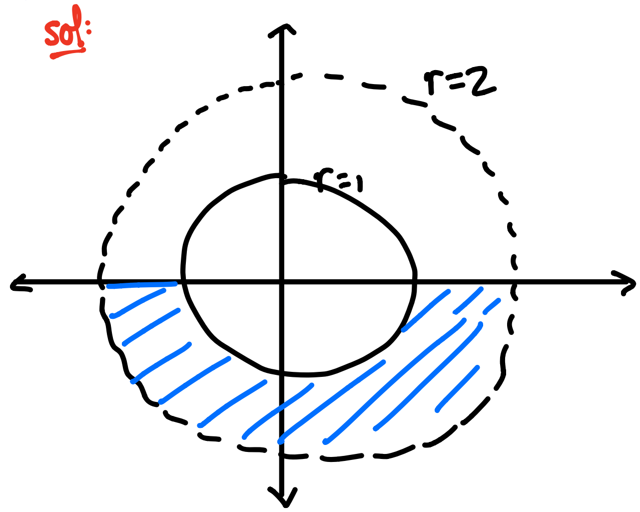
$\boxed{A}$ $r \le 1$, $\pi/2 \le \theta \le 3\pi/2$

Sol:



$\boxed{B}$ $1 \le r < 2$, $\pi \le \theta \le 2\pi$

Sol:



PART 4: SLOPE AND TANGENT LINES

** Suppose we have the following function in POLAR COORDINATES: $r = f(\theta)$

θ = PARAMETER

! dy/dx = SLOPE

$\begin{cases} y = r\sin\theta \\ x = r\cos\theta \end{cases}$ fn of θ

$\boxed{\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}}$ ← USE THIS

GENERAL SLOPE $= \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{\text{PROD. RULE}}{\text{PROD. RULE}}$

THE EQUATION of A TANGENT LINE:

$y - y_1 = m(x - x_1)$

$m = \dfrac{dy/d\theta}{dx/d\theta}$

Ex5: Find the equation of the **TANGENT LINE** to the graph of $r = 3\sin(2\theta)$ when $\theta = \pi/3$.

Sol:

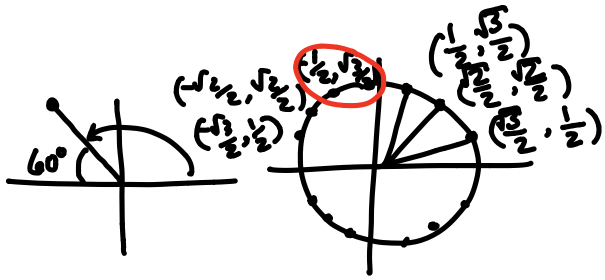
$$y = r\sin\theta = 3\sin(2\theta)\cdot\sin\theta$$

$$x = r\cos\theta = 3\sin(2\theta)\cos\theta.$$

POINT: @ $\theta = \pi/3$

$$y = 3\sin(2\pi/3)\,\sin(\pi/3) = 3\,\tfrac{\sqrt{3}}{2}\cdot\tfrac{\sqrt{3}}{2} = \boxed{\tfrac{9}{4} = y_1}$$

$$x = 3\sin(2\pi/3)\cos(\pi/3) = 3\tfrac{\sqrt{3}}{2}\cdot\tfrac{1}{2} = \boxed{\tfrac{3\sqrt{3}}{4} = x_1}$$



SLOPE: @ $\theta = \pi/3$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\sin(2\theta)\cos\theta + 6\cos(2\theta)\sin(\theta)}{-3\sin(2\theta)\sin\theta + 6\cos(2\theta)\cdot\cos\theta} \quad @\ \pi/3.$$

$$= \frac{3(\sqrt{3}/2)(1/2) + 6(-1/2)\cdot(\sqrt{3}/2)}{-3(\sqrt{3}/2)(\sqrt{3}/2) + 6(-1/2)(1/2)} = \frac{\cancel{3/2}}{\cancel{3/2}}\cdot\frac{(\sqrt{3}/2 - \sqrt{3})}{(-3/2 - 1)}$$

$$= \frac{(\sqrt{3}/2 - \sqrt{3})}{-5/2}$$

$$= \frac{-\sqrt{3}/2}{-5/2} = \boxed{\tfrac{\sqrt{3}}{5} = m}$$

TANGENT $\quad y - y_1 = m(x - x_1)$

$$\boxed{y - 9/4 = \frac{\sqrt{3}}{5}\left(x - \frac{3\sqrt{3}}{4}\right)}$$