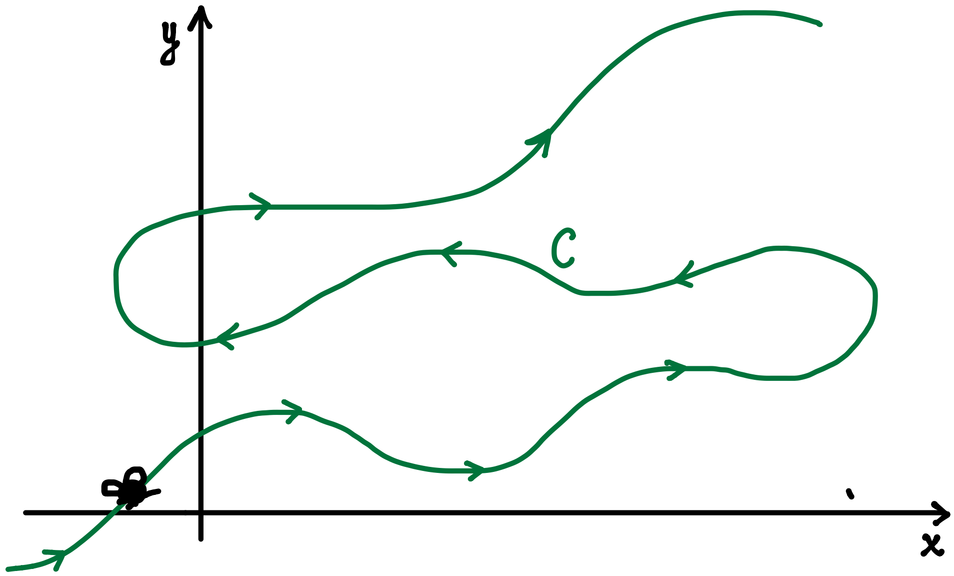
CH 10.1 PARAMETRIC EQUATIONS

MOTIVATION



** Consider a particle moving along the curve C in the indicated direction. We can't describe C as a function y=f(x) since the curve does not pass the vertical line test.... But we can still describe the motion of the particle using what we call PARAMETRIC EQUATIONS.

PART 1: THE BASICS

Ex 1: Sketch the curve defined by the **PARAMETRIC EQUATIONS**:

$$x = t^2 + 2t \quad \text{and} \quad y = t + 1$$

Sol:

1

DESMOS DEMO: Let's look at some awesome curves that are defined parametrically.

Ex 1:

$x = t + \sin(5t)$
$y = t + \sin(6t)$

Ex 2:

$x = (\sin(7\pi t))^3$
$y = (\cos(5\pi t))^3$

Ex 3:

$x = 2.3\cos(10t) + \cos(23t)$
$y = 2.3\sin(10t) - \sin(23t)$

Ex 2. Consider the following PARAMETRIC CURVE:

$$(x,y) = (t-1,\ 4t) \qquad 0 \le t \le 2$$

A Write x in terms of y (I.e. eliminate the parameter t)

Sol:

B Draw a picture of the PARAMETRIC CURVE and clearly indicate the initial and terminal points as well as the direction of increasing values of "t".

Sol:

Ex 3. Consider the following PARAMETRIC CURVE:

$$(x,y) = (t^2,\ 2t^3) \qquad 0 \le t \le 1$$

A Write x in terms of y (I.e. eliminate the parameter t)

Sol:

B Draw a picture of the PARAMETRIC CURVE and clearly indicate the initial and terminal points as well as the direction of increasing values of "t".

Sol:

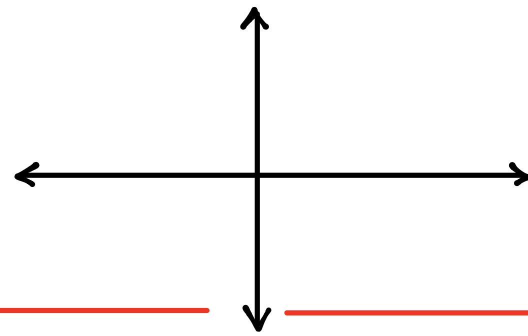
PART 2: ROTATIONAL PARAMETRIC CURVES

THE UNIT CIRCLE

* We can express the unit circle using parametric equations!

$(x, y) = (\cos(t), \sin(t)) \quad 0 \leq t \leq 2\pi$

t	x	y



TRIG IDENTITIES

$\cos(-t) = \cos(t)$

$\sin(-t) = -\sin(t)$

$\cos(t + \pi/2) = -\sin(t)$

$\cos(t + \pi) = -\cos(t)$

$\sin(t + \pi/2) = \cos(t)$

$\sin(t + \pi) = -\sin(t)$

NOTE: Using the parametric representation of the unit circle... We can find parametric equations for circles with other properties!

REVERSE DIRECTION

CENTER (h,k) RADIUS "r"

UNIT CIRCLE

$(\cos(t), \sin(t))$

$0 \le t \le 2\pi$

CHANGE INITIAL POINT

FASTER ROTATION

Ex 4. Find a PARAMETRIZATION of a circle centered at the origin with radius 3 that satisfies the following:

- Counterclockwise orientation with initial point (3,0)

sol:

- Clockwise orientation with initial point (3,0)

sol:

- Counterclockwise orientation with initial point (0,3)

sol:

- Clockwise orientation with initial point (0,3)

sol:

Ex 5. Find the PARAMETRIZATION of a circle with center (1,2), radius 5, oriented clockwise, with initial point (-4,2).

sol:

Ex 6. A fly moves along a circle $(x-1)^2 + y^2 = 25$

Find a PARAMETRIC CURVE that describes that path of the fly in each of the following cases:

- The fly travels in a clockwise direction starting at (5,0) with $0 \le t \le 2\pi$

sol:

- The fly travels halfway around the circle in a counterclockwise direction starting at (1,5) with $0 \le t \le \pi$

sol:

Ex7: Two particles travel in space according the following **PARAMETRIC EQUATIONS:**

$$(x_1, y_1) = (5\sin(t), 2\cos(t)) \qquad 0 \leq t \leq 2\pi$$
$$(x_2, y_2) = (-5+\cos(t), 1+\sin(t))$$

Find all points of intersection of the two paths and then find the location of any **COLLISION POINTS** (if they exist).

sol: