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ABSTRACT
The design and initial implementation of a project-based directed study course in mathematical research and communication is described. The intention of the course is to expose students to the foundation of mathematical research through inquiry-based learning. Students are asked to approach an exercise in topology as though it is an open problem. At the end of the semester, students are also required to prepare and deliver a presentation on their work from the semester.

KEYWORDS
Mathematics; inquiry-based learning; project-based learning; undergraduate research; directed study

1. INTRODUCTION

It is an unfortunate reality that many students learning mathematics often see it as a fixed subject, holding the erroneous belief that there is nothing new to discover within the field. What do people actually research? What is left to learn? How does one go about trying to uncover something valuable or novel?

As we know, this notion could not be further from the truth. Mathematics is an invaluable tool that allows us to investigate our physical world and explore new and unforeseen avenues. To help students make the transition to upper-level mathematics and to become acquainted with mathematical research, many programs offer student research opportunities. These opportunities range from Research Experiences for Undergraduates (REUs) to thesis projects and independent (or directed) studies. Countless studies have been conducted related to the effectiveness of such experiences. In [7], the authors focused on instructor and student perspectives related to directed studies (DS courses). They found that

> For students, DS courses enhanced several core academic and research skills, and for instructors, DS courses provided opportunities for collaborative research with students and generated energy and enjoyment.

Their conclusions are in agreement with conclusions found in [3, 4, 13] regarding the effectiveness of these types of research experiences. The widely held belief is that undergraduate students participating in research experiences take developmental strides that cannot be replicated through traditional, instructor-led courses. The

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authors in [6] have compiled a useful resource to help faculty implement REU's effectively.

Independent study courses and research experiences are also aligned with current pedagogical practices that emphasize less direct instruction and more student inquiry. Common practices include discovery learning, active learning, and project-based learning, which all fall under the category of inquiry-based learning (IBL). In [10], the author describes *inquiry-based learning* in the following way:

> Inquiry-based learning is characterized by the fact that learners shape, learn and deliberate on the process of a research project [...] doing so throughout all the essential phases of said project; from developing questions and hypotheses, selecting and implementing the methods, through testing and presenting the results.

There is a plethora of research that has been conducted regarding the implementation of IBL. An innovative project-based framework is presented in [9] and further strategies for implementing IBL in STEM disciplines are provided in [1]. Research also strongly supports the efficacy of IBL. The authors in [8] share the long-term results of IBL within a college mathematics class. They found that active learning strategies had a "lasting and significant" effect on students of all ability levels.

The authors of the current article set out to develop a course based on an independent study model, while also incorporating key facets of IBL. The main goal was to broaden student perspectives on mathematical research while also allowing them to become exposed to concepts from an unfamiliar field of mathematics. As part of this class, the intention was also to provide students with an opportunity to communicate their ideas orally and in writing. In order to do this, the authors developed a *course format* that can be applied to various mathematical content areas and focuses on *mathematical research and communication*. For the remainder of this paper, this will be referred to as an "MRC" format course.

The research experience provided by a course in the MRC format is simulated. Students are not working on an open problem, which serves two main purposes: it relieves pressure and anxiety on the student knowing that their problem is standard to the field they are studying, and it allows the instructor the time and energy to take on more students at one time than they generally could with a standard REU. The evidence suggests that such an experience is beneficial either as preparation for a standard REU, or in the lack of such an opportunity, a replacement thereof.

An initial version of a course in this format was run at United States Military Academy (USMA) in the fall 2019 semester under the title *Introduction to Topology*, with the first author as the instructor. The course was run as an independent study with four USMA students. One of the four took the course remotely while studying abroad at a foreign military academy for the semester. Despite the title of the course and the fact that the material studied was point set topology, the main focus of this article will be on the MRC format of the course. The coming sections include (i) an overview of the MRC format, (ii) information related to the design of a course in this format, (iii) observations related to the first implementation of the course, and (iv) conclusions that were drawn upon reflecting on the experience from both the instructor's and students' perspectives.

2. DESIGN OF THE COURSE

In order to successfully offer a course of this type, one must carefully design and construct the course based on the primary goals and objectives of the course as well as the intended students who would be interested in taking the course.

2.1. Goals and Objectives

The course was designed around the following primary goal:

Goal 1: Students will be exposed to the basic facets of mathematical research and communication.

The intention is for students to have low stakes exposure to the process of mathematical research in order to gain an understanding of what academic research involves, particularly in the area of pure mathematics. This experience is designed to (i) prepare students to engage in undergraduate research opportunities within the broader academic community and (ii) prepare students for academic endeavors after completion of their undergraduate program (i.e., graduate school, institutional research, etc). Ultimately, students should be engaged in such a way that their curiosity and thirst for knowledge will be ignited.

To accomplish this primary goal, the course designers included the following course objectives:

Objective 1: Students will solve a problem in an unfamiliar mathematical field by successfully planning and executing a self-directed and self-paced learning experience.
Objective 2: Students will effectively communicate mathematical reasoning through the creation of an expository piece of writing.
Objective 3: Students will effectively communicate mathematical reasoning orally through the delivery of a professional presentation.

The expected outcome of the course was for each student to produce a self contained paper which begins by presenting all of the background information necessary to solve an exercise chosen from an approved list taken from the textbook, [12], and concludes with the solution to that exercise. Through the completion of this project and presentation of their results, students would successfully meet the three key objectives and accomplish the primary goal of the course. The students were directed to treat this exercise as a research project on an open problem and to receive any assistance necessary on the final proof only from the instructor. This was easily enforceable at USMA, since school policy requires every assignment completed outside of class and turned in for a grade to have a cover sheet signed by the student stating that they have properly documented any assistance received. Failure to properly document anything could result in a violation of the honor code, which could then lead to separation from the academy. The papers were expected to be written with an intended audience of individuals who have just completed a basic course in mathematical proof.

2.2. The Curriculum

Unlike most instructor-led courses, a course offered in the MRC format does not have a set curriculum. The course can focus on any topic within the discipline. The instructional design process simply involves (i) selecting an overall topic/textbook and (ii) compiling a list of potential projects for students to choose from.

Topic/Textbook Selection: It is critical to choose an appropriate topic for the course. The topic must be broad enough so that all students can work on a project that falls under that topic umbrella, and it must be chosen so that students have had limited exposure to it in their mathematical backgrounds. It is important that students come into the course with nearly a blank slate on which they can build their own foundation. Topics can vary depending on the academic field and the level of students enrolled in the course. For the math MRC course offered at USMA, students were third- and fourth-year mathematics majors. Based on the interest of the instructor, Topology was chosen as the central topic. Other mathematical concepts that would serve as good central topics for an MRC course are discrete chaos, mathematical modeling, number theory, cryptography, etc.

Once a topic is chosen, it is important to pick a textbook that will fit with a course of this type. Textbooks should offer a comprehensive analysis of the central topic and should include an assortment of exercises from which students can focus on for the semester. Since the textbook will be the main source of information for the student, care should be taken to choose an appropriate text.

Project Selection: One of the most important pieces of the design of the course is choosing exercises from the textbook which are appropriate for having students work on for a full semester. Since their entire experience with the material will be focused on developing the background for solving this one problem, it is imperative that this exercise requires background in as many of the foundational topics of the subject as possible. However, this desire to cover many important topics needs to be balanced with the timeline of having a single semester to complete the paper and prepare a presentation on it. The following is the list of exercises from [12] that the students were given to choose from in the first iteration of the course:

> § 22, **Problem** 3: Let $\pi_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be projection on the first coordinate. Let A be the subspace of $\mathbb{R} \times \mathbb{R}$ consisting of all points $x \times y$ for which either $x \geq 0$ or $y = 0$ (or both); let $q : A \to \mathbb{R}$ be obtained by restricting π_1. Show that q is a quotient map that is neither open nor closed.

§23, **Problem** 11: Let $p : X \to Y$ be a quotient map. Show that if each set $p^{-1}(y)$ is connected, and if Y is connected, then X is connected.
§24, **Problem** 8: (a) Is a product of path-connected spaces necessarily path connected? (b) If $A \subset X$ and A is path connected, is $\overline{A}$ necessarily path connected? (c) If $f : X \to Y$ is continuous and X is path connected, is $f(X)$ necessarily path connected? (d) If $\{A_\alpha\}$ is a collection of path connected subspaces of X and if $\bigcap A_\alpha \neq \emptyset$, is $\bigcup A_\alpha$ necessarily path connected?
§25, **Problem** 2: (a) What are the components and path components of $\mathbb{R}^\omega$ (in the product topology)? (b) Consider $\mathbb{R}^\omega$ in the uniform topology. Show that $\mathbf{x}$ and $\mathbf{y}$ lie in the same component of $\mathbb{R}^\omega$ if and only if the sequence

$$\mathbf{x} - \mathbf{y} = (x_1 - y_1, x_2 - y_2, \ldots)$$

is bounded. [*Hint*: It suffices to consider the case, where $\mathbf{y} = \mathbf{0}$.] (c) Give $\mathbb{R}^\omega$ the box topology. Show that $\mathbf{x}$ and $\mathbf{y}$ lie in the same component of $\mathbb{R}^\omega$ if and only if the sequence $\mathbf{x} - \mathbf{y}$ is "eventually zero." [*Hint*: If $\mathbf{x} - \mathbf{y}$ is not eventually zero, show there is a homeomorphism h of $\mathbb{R}^\omega$ with itself such that $h(x)$ is bounded and $h(y)$ is unbounded.]
§26, **Problem** 8: Let $f : X \to Y$; let Y be compact Hausdorff. Then f is continuous if and only if the *graph* of f,

$$G_f = \{x \times f(x) \mid x \in X\}$$

is closed in $X \times Y$.
§26, **Problem** 12: Let $p : X \to Y$ be a closed continuous surjective map such that $p^{-1}(\{y\})$ is compact, for each $y \in Y$. (Such a map is called a *perfect map*.) Show that if Y is compact, then X is compact. [*Hint*: If U is an open set containing $p^{-1}(\{y\})$, there is a neighborhood W of y such that $p^{-1}(W)$ is contained in U.]
§29, **Problem** 6: Show that the one-point compactification of $\mathbb{R}$ is homeomorphic with the circle S^1.
§33, **Problem** 2:[1] (a) Show that a connected normal space having more than one point is uncountable. (b) Show that a connected regular space having more than one point is uncountable.

In generalizing the MRC course format to another topic, problem selection is the key to the success of the course. A problem should be chosen based on the solution requiring as many of the key concepts as possible which would typically be included in a standard undergraduate course in the subject. Throughout the process of solving their problem, each student will undoubtedly engage with concepts which end up not being necessary for them; however, it is important to ensure that each student's path of least resistance remains sufficient for credit for a course in the subject

[1] The problem from §33 requires the Urysohn Lemma, a very deep result. It was specified in the document provided to the students with the exercise choices that the proof of this Lemma would not be required to be included in the final paper for this problem.

area. Once the project exercises are narrowed down, a document can be created to provide to the students. They will then be able to choose their own project exercise based on the descriptions provided and on advisement of the instructor. Ideally, the problem statements should initially sound completely foreign to them. This makes the entire process of discovery more rewarding. For more tips and advice on choosing topics for student projects, see [11].

2.3. Structure of the Course

The course has four main components; (i) weekly meetings, (ii) checkpoints, (iii) final paper, and (iv) final presentation. These four components also served as the four major graded components within the course.

Weekly Meetings: Students were asked to arrange a meeting with the instructor each week of the semester. During these meetings, students were expected to be ready to describe in detail what they had learned since the previous meeting. They were also given an opportunity to ask questions on anything they were not understanding. These meetings generally lasted anywhere from 20 min to 1 h.

Checkpoints: Four checkpoints were scheduled throughout the semester, wherein students were asked to submit their paper in its current state at that time. The intention of the checkpoints was to make sure that students stayed on pace to finish the paper in the time allotted, as well as to provide detailed guidance on style and accuracy in their writing along the way. At the fourth checkpoint, scheduled approximately a month before the end of the semester, students were expected to turn in the first draft of a completed paper.

Final Paper: The largest graded event was the final paper. Students were expected to illustrate various concepts with original examples not contained in the assigned text. They were asked to provide proofs in their own words for all statements taken from the text, whether proven there or not. The last section of the paper was expected to contain a valid proof for the assigned problem, representing the author's own work. Students were also asked to provide some type of further discussion on their result. Suggestions were provided to this end, including identifying and proving a corollary, analyzing hypotheses, an interesting example illustrating the result, or some other creative discussion.

Presentation: The final graded event of the course was a presentation on the work done throughout the semester. Each student was asked to schedule and deliver a 45 min presentation on their project during the last week of classes. All students were expected to attend every presentation, to the extent that this was possible. Students were instructed to direct their presentation to an intended audience of individuals having very basic familiarity with the subject of topology, such as what you would get from a typical course in real analysis.

2.4. Target Students

It is important to note that a course offered in this format is intended for a specific type of student and that prospective students must be made aware of the structure of the class as well as the hardships that they are likely to face. The authors in [5] explore how to determine if research experiences are right for a given student. Ultimately, it is beneficial for students to have a *growth mindset*. In [2], Carol Dweck describes a growth mindset in the following way:

> In a growth mindset, people believe that their most basic abilities can be developed through dedication and hard work; brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment.

An MRC formatted course is different from a standard mathematical course. There is no set procedure for students to follow and no computations that can merely be mimicked. Instead, students need to genuinely engage with mathematical concepts that are unfamiliar to them. They need to plan their own course of action and outline their own educational experience. During that process, they will surely enter a state of disequilibrium in which they may feel lost and frustrated. Students must be able to recognize that these feelings are commonplace within the overall research process and necessary for gains to be made. They must understand that the hardships will make them stronger and that finding their way out, largely on their own, will contribute to a stronger and deeper understanding of the fundamental principles that they are studying. This is a growth mindset and it is an important skill that will help students enjoy and succeed in such a course.

The format of the course and the potential rigors that students may face should be clearly communicated to them when the course is advertised amongst the math majors.

At USMA, there were four students enrolled in the course in fall of 2019. All of these students had heard about the course while taking real analysis the previous semester with the instructor and they were intrigued by the format. The three students taking the course normally were in the first semester of their firstie (senior) year, while the one studying abroad was starting cow (junior) year. All firsties are required to complete a capstone research project for graduation from USMA, and part of the reasoning behind running this course was to provide some practice to help this project go more smoothly. Nonetheless, the four students participating in the class were highly motivated and fully understood that much of the work would be self-guided and that they would naturally experience hardships throughout the experience.

3. IMPLEMENTATION

In this section, observations are presented related to the first implementation of the MRC course at USMA.

Preparation: In the week leading up to the fall semester, students were sent an email containing some of the course documents to give them an idea of how the course would be run. In this email, students were asked to schedule a meeting with the instructor during the first week of classes. At these initial meetings, there was a quick discussion of the expectations for the students, and the list of exercise choices was provided. Students were instructed to provide an ordered list of their top four choices when they were ready, and that they would get their first choice unless it was already taken. There was no deadline provided; the only incentive to choosing quickly was getting their first choice, and it turned out that all four students got their first choice. The last list of choices was submitted before the end of the first week of classes, so the students did not take much time to really understand what they were choosing, which was not contradictory to the intention of the instructor. Of the eight problems on the list, the four that were chosen were §22 Problem 3, §26 Problem 8, §29 Problem 6, and §33 Problem 2.

Entry Survey: Also at this first meeting, the students were asked to fill out a survey in the first week of classes to assess their attitudes toward the course coming in. There was one student who was uninterested in developing his oral communication of mathematical results. All students were interested in developing their proof and mathematical writing skills, as well as learning the topology material. None of the students expected the course to be easy, and only one expected to spend less time on it than a standard course. All students were very excited to have the freedom to work on their own schedule. There was one student who came in with some confidence in his ability to perform mathematical research ; however, this same student was not confident in his ability to communicate mathematical results, either orally or in writing. Another student did not express particular confidence in performing research, however, was very confident in communicating mathematics. There were two students with plans to attend graduate school in mathematics or a related field, one who was unsure, and one who had no such plans.

Instructor/Student Meetings: Throughout the semester, students were very consistent about properly scheduling their weekly meetings with the instructor. There was significant overlap in the topics discussed in these meetings early in the semester. At this time students were working through the basics of topology before being able to focus their attention on their specific problem and the topics more closely related to its solution. Once the students had the background to approach their problems, meetings became very specialized and focused mostly on solving the chosen problem, with occasional discussions on the writing they were working on and comments they had received on checkpoints. All of the students were able to solve their problems with only gentle guidance from the instructor where necessary.

Major Checkpoints: The four checkpoints were scheduled very carefully throughout the semester. The first was at the start of the fourth week of classes. This came very fast for students, with the intention of getting them writing early in the process. This had a two-fold benefit. First, it eliminated the possibility of procrastination due to an overall very busy schedule. Second, writing forced them to truly take on the concepts and really think about them rather than reading and convincing themselves they understood better than they did. At the time of the first checkpoint, it was only expected that there be some basic foundational concepts discussed, such as the definitions of a topology, an open set, and a basis.

The next two checkpoints were spaced approximately sixteen days apart from the previous. Around the time of the third checkpoint was when the students were all just beginning to truly attack their problems, having gotten through the necessary background. More than three weeks were provided before the final checkpoint, at which it was expected that the students would turn in a completed first draft of their paper. In practice, the students were mostly done with their solutions at this time, however there were still loose ends to be tied up in both the final proofs as well as the background information. There was no concern that the papers were not in completed first draft form at this time.

Distance Learning: The student who was studying abroad certainly had a different experience with the course than the other three. First, he had a late start, as his host academy was immersed in extensive military training for the first few weeks of the USMA academic year. Regular communication of some form with the instructor was required. Much of this communication occurred through email updates with some basic questions on what had been read. Occasionally, video chat meetings were conducted in order to clear up confusions that were more difficult to get at through email. This student was not held to the same checkpoint schedule as the others. It was around the time of the last checkpoint when his first submission was made, and the paper was close to completed first draft form at that time. There were two further drafts submitted to the instructor for which feedback was provided before the final paper submission was made.

Final Papers and Presentations: The final papers ranged from 11–15 pages in length. Though not fully polished due to the single semester timeline, all of the papers were representative of very high quality undergraduate work. Some of the students came in with well honed proof writing skills from previous courses, and they learned a lot throughout the semester about structure, organization, and dialogue connecting their results. With guidance, these skills developed drastically for all students and could be considered the largest area of success from the course.

Each of the three students at West Point delivered a presentation on their work around the last week of classes. The audiences consisted of the remaining two students, the instructor, and a handful of faculty members of the West Point Department of Mathematical Sciences. Each presentation lasted approximately 1 h, with about 45 min spent on the prepared work and the remaining 15 min on questions from the audience. A thorough understanding of the basic concepts of topology as well as the particular problem solved was clearly conveyed by each of the students in their presentation.

4. DISCUSSION AND CONCLUSION

To determine the efficacy of the MRC formatted *Introduction to Topology* course, the authors reflected on several key elements including (i) content coverage, (ii) student perceptions, and (iii) possible improvements.

Papers	4	3	2
Topic	Open Sets Closed Sets Basis Hausdorff Subspace Continuity	Closure Homeomorphism	Compactness Sequences Interior Limit Point Product Space Projection Open/Closed Maps

Figure 1. Some basic topology concepts and the number of student papers in which the topic was covered.

Content Coverage: While the primary intent of the course was to teach mathematical research and communication, the course was run under the title *Introduction to Topology*, and the students registered for the course to learn topology. One concern going into the course was that exposure to many of the fundamental concepts in the field could be limited by this novel approach. As it turned out, many of the concepts which would be covered in a standard undergraduate course in topology appeared in more than one of the papers, see Figure 1. It was also clear that in the process of working their way toward a solution to their problems, each of the students deeply explored many concepts which did not find their way into the paper in the end. There is no concern that the students missed out on anything they would have gotten from a standard course. Moreover, the students meaningfully learned the topological concepts and are more prepared to apply this knowledge in future academic endeavors.

Topics which were included in only one of the papers were Lindelöf spaces, connectedness, regular and normal spaces, the metric topology, compactification, and quotient maps.

Student Perceptions: After the last presentation was given, students were asked to complete a final survey on their experience with the course. The results of this survey were overall very positive. All students strongly agreed that they had learned a lot about topology and that their skills related to both mathematical proof and mathematical communication were significantly developed. None of the students found the course to be easy, and only one found that it required less time than most other courses. The freedom to work on a flexible schedule was greatly appreciated. Confidence in performing basic mathematical research as well as precisely communicating mathematical results both orally and in writing was significantly increased across the board amongst the students.

Students were reminded that the main goals of the course were to introduce them to the process of performing mathematical research and to develop their precise communication of mathematics both orally and in writing. When asked to comment on how well these goals were met throughout the semester, the responses were:

> During this course, I had to learn the course material by myself and decide what I wanted to write about along with how I wanted to organize my final product.

> ...this course greatly enhanced my ability to do mathematical research as well as communicate in general.

> Both of these goals were met for me by taking this course. I became a lot more proficient at reading and practicing mathematics on my own, which is essential for research.

> ...the paper and presentation requirements gave me a more realistic expectation of how much work goes into preparing a product.

> The autonomous structure of the course enabled me to attack my problem independently and learn how to independently study mathematics.

> I thought these goals were met very well.

Possible Improvements: In the process of reflecting on the first iteration of a course in the MRC format, there are a few things that could be improved upon.

First, one of the students suggested that the instructor allow more time to learn about each of the possible problems before requiring students to make a choice. While they were not given a deadline to choose, they felt the need to choose quickly to not have their first choice taken by another student. There is some concern of throwing off the timeline of the entire semester by allowing them more time to choose, with two potential ways to alleviate this concern. At the first checkpoint, all of the papers were looking very similar, as the students were only getting through the very basics of the material which were sure to be important to the final papers. It is not necessary to require a choice until after this time. Also, the list of students taking the course is generally set well in advance of the semester, so the problem choices could be provided in the weeks leading up to the start of the course. It would then be up to the students to do some pre-reading if they want to make a more knowledgeable choice of project.

Further, weekly student meetings could be more structured. The questions were almost always directed toward content, and students less often had interest in discussing the writing that they were doing, which limited their writing feedback mostly to the four checkpoints. It could be an improvement to stipulate from the start that each meeting will consist of discussion of content, as well as discussion of writing.

Finally, students could be provided with more opportunities to develop their formal oral communication of mathematics. Throughout the semester, students communicated their findings informally with the instructor at least weekly. Their only opportunity for formal oral communication was the presentation at the end of the semester. It would be beneficial to have a time when all parties are available each week and to assemble concomitant with each checkpoint and have each student take a short time to present their current work in a formal way to the instructor and other students.

Ultimately, the first iteration of the MRC formatted course at USMA was a success and the authors are excited to continue to offer courses of this type and to refine the learning experience for students. The authors feel as though this model can be applied on larger scales and in a vast assortment of disciplines. Within undergraduate math programs specifically, a course of this type should be available as an option for all students pursuing an undergraduate degree in mathematics and strongly encouraged for those considering graduate education. This type of course gives students an opportunity to apply the reasoning skills that they have developed during their undergraduate careers to "test the waters" themselves and triggers developmental strides that otherwise may not have been possible.

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BIOGRAPHICAL SKETCHES

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David McArdle earned his Ph.D. in mathematics at the University of Rhode Island in 2017 and is now an assistant professor in residence at the University of Connecticut. His primary research is in the field of discrete dynamical systems, but he also has a strong interest in improving math education at the collegiate level.